# BOHR'S ATOM - (The Hydrogen-Like Atom)

Postulates:

1. Quantized angular momentum,

(1) $$mv_n r_n = n\frac{h}{2\pi}, \qquad n = 1, 2, 3, \ldots$$

m = mass

$v_n$ = linear velocity, $n^{th}$ level

$r_n$ = radius, $n^{th}$ level

h = Planck's Constant = $6.6260755 \times 10^{-34}$ J·s

**2.** Electrons move in circular orbits.

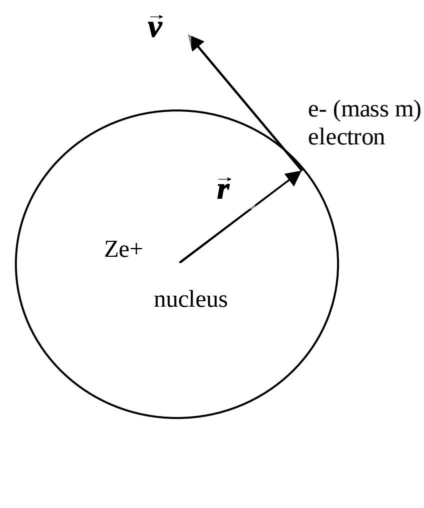
3. Electron doesn't radiate while in orbit.

By balancing the centripetal force (toward the center) , due to electrostatic attraction $\left(= Ze^2 / r_n^2\right)$ (where Z = number of protons and e = electronic charge = $1.60217733 \times 10^{-19}$ C) with the centrifugal force, due to acceleration in a rotating frame, (away from the center) $\left(= -mv_n^2 / r_n\right)$ (Note that m is approximately the mass of the electron, $m_e = 9.1093897 \times 10^{-31}$ kg, however, one should use the "reduced mass", but we will ignore that at present.) the equation of motion for an electron in orbit can be written as:

(2) $$\frac{Ze^2}{r_n^2} - m\frac{v_n^2}{r_n} = 0$$

***or, rewriting:***

$$\frac{Ze^2 r_n}{m} = \left(r_n v_n\right)^2$$




Substituting from eqn. (1) above:

$$\frac{Ze^2 r_n}{m} = \left(\frac{nh}{m \cdot 2\pi}\right)^2$$

Solving for $r_n$:

(3) $$r_n = \frac{1}{Z}\left(\frac{h^2}{me^2 \cdot 4\pi^2}\right)n^2 = \frac{an^2}{Z}, \qquad a = 0.53 \times 10^{-8}\,cm = 0.53 \text{ A } (\textbf{for hydrogen } Z = 1)$$

The energy of the system is given by:

$$(4) \qquad E_n = \tfrac{1}{2} m v_n^2 - \frac{Ze^2}{r_n}$$

By rearranging the equation of motion (eqn 2.) above to give***:***

$$m v_n^2 = \frac{Ze^2}{r_n}$$

Which can then be substituted into eqn. 4 for the energy of the system, giving:

$$E_n = -\frac{Z^2 e^2}{2 r_n}$$

Finally, substituting for $r_n$ from eqn. 3 gives:

$$E_n = -\frac{Z^2 e^2}{2}\left(\frac{4\pi^2 m e^2}{h^2 n^2}\right) = -Z^2 \frac{2\pi^2 m e^4}{h^2} \cdot \frac{1}{n^2} = \frac{Z^2 E_1}{n^2}$$

$$E_1 = -\frac{4\pi^2 m e^4}{2h^2} = 2.179 \times 10^{-18} J = -13.595 eV \quad (\textit{for hydrogen}, Z = 1)$$

For photons (E = hν) associated with transitions (absorption, emission) one looks at the absolute value of the difference between energy levels of the hydrogen atom:

$$h\nu_{mn} = E_m - E_n = Z^2 |E_1| \left(\frac{1}{n^2} - \frac{1}{m^2}\right), \qquad (n < m)$$

or, since $1/\lambda_{mn} = \nu_{mn}/c$

$$\lambda_{mn}^{-1} = Z^2 \frac{|E_1|}{hc}\left(\frac{1}{n^2} - \frac{1}{m^2}\right), \qquad (n < m)$$

$$\text{Rydberg's constant} = \frac{|E_1|}{hc} = R_\infty = 1.0973731 \times 10^5 \, cm^{-1}$$

Emission Line Series (Hydrogen Atom, Z = 1)

| | | |
|---|---|---|
| ***Lyman*** | ***m>1 to n = 1*** | ***Ultraviolet*** |
| ***Balmer*** | ***m>2 to n = 2*** | ***Visible*** |
| ***Paschen*** | ***m>3 to n = 3*** | ***Infrared*** |
| ***Brackett*** | ***m>4 to n = 4*** | ***Infrared*** |
| ***Pfund*** | ***m>5 to n = 5*** | ***Infrared*** |

A sometimes useful relationship:

$$\lambda(\text{Å}) \Leftrightarrow 12398 / E(\text{eV})$$

so for $E_1$ = 13.595 eV:

$$\lambda_1 = 912 \text{ Å} = 91.2 \text{ nm}$$

**A note on units:** The above formulation is done using Gaussian units (cm, g, s) rather than MKS (m, kg, s) and this impacts the units since the constants for space associated with electromagnetic behavior are different. (These constants, the electric permittivity, $\epsilon_0$, and magnetic permeability, $\mu_0$, in vacuum and $\epsilon$ and $\mu$ in a non-vacuum medium connect to the electric and magnetic fields by Maxwell's equations.) In the Gaussian system $\epsilon_0 = 1$ and $\mu_0 = 1$ giving the potential energy between two point charges as $W = (q_1q_2)/r$, while in the MKS system $\epsilon_0 = (1\times10^7)/(4\pi c^2)$ and $\mu_0 = 4\pi \times 10^{-7}$ giving the potential energy between two point charges as $W = (1/4\pi\epsilon_0) \times (q_1q_2)/r$. (Charge in the Gaussian system is in statcoulombs and is in coulombs, C, in the MKS system – (1 C = $N_A \times e = 3\times10^9$ statcoulombs))