

EVALUATING WORD PROBLEMS USING POLYA'S PROBLEM-SOLVING STRATEGY:
DETERMINING ITS EFFECTS ON AN ENGLISH LANGUAGE LEARNER'S WRITTEN
AND ORAL COMMUNICATION

by

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To Christopher

In appreciation of your assistance, patience, and support.

TABLE OF CONTENTS

Chapter One: Introduction	1
Chapter Two: Literature Review	12
Recent Theories on Mathematics Instruction and Assessment	12
The Language of Math	15
Relating Speaking and Writing to Mathematics.....	20
Polya’s Problem-Solving Plan.....	23
Schema-Based Instruction.....	24
Strategies Used by Proficient Problem Solvers.....	25
The Gap	27
Research Questions	28
Summary	29
Chapter Three: Methodology.....	30
Mixed Method Research Paradigm	31
Data Collection.....	32
Procedure.....	34
Data Analysis	39

Triangulation	41
Ethics	41
Conclusion.....	42
Chapter Four: Results	43
MCA-II Practice Problems	43
Retrospective Reports.....	45
Fluency	53
Vocabulary and Content Knowledge	56
Conclusion.....	58
Chapter Five: Conclusions.....	59
Major Findings	59
Implications	64
Limitations.....	66
Further Research.....	67
Appendix A.....	72
Appendix B.....	77
Appendix C.....	84
Appendix D.....	87
References.....	123

LIST OF FIGURES

Figure 4.1 MCA-II Constructed Response Average Scores	45
Figure 4.2 Polya's Problem Solving Plan Average Scores	46

LIST OF TABLES

Table 2.1 Polya’s Problem-Solving Plan.....	24
Table 3.1 Polya’s Problem-Solving Plan Checklist.....	34
Table 3.2 MCA-II Sample Rubric	35
Table 3.3 MCA-II Sample Constructed Response Question	37
Table 3.4 Data Collection Schedule.....	38
Table 4.1 Numbered Coding of Polya’s Steps.....	47
Table 4.2 Polya Steps Used by Participants in Retrospective Reports	49
Table 4.3 Fluency Test: Number of Pauses in Retrospective Reports.....	53

CHAPTER ONE: INTRODUCTION

The United States has become focused on achieving success for all students including the growing number of English Language Learners (ELLs) in our public schools. With the introduction of No Child Left Behind (NCLB, 2001), many educators feel pressure to find the best methods and strategies for teaching in order to help all students pass the required state tests. In Minnesota, the Minnesota Comprehension Assessment II (MCA-II) tests the majority of students in mathematics and reading. All students must take these exams in English, even when English is their second language. All students are asked to explain their thinking in writing on the constructed response or essay portions of the MCA-II math and reading tests. For ELLs, this is a difficult task because they are still developing their English-language skills. Because these students still struggle with language, the MCA-II math test is an assessment of both language and math ability.

As a middle school teacher of ELLs, I was hired to work in collaboration with mathematics teachers to teach both sheltered and co-taught mathematics. In a collaborative classroom, a content and English as a Second Language (ESL) teacher work together to write lesson plans that effectively address the content and language needs of their native and non-native English speakers. Our district added these classes in order to support our ELLs and to improve our MCA-II test scores. After three years of

observation and teaching, I became aware of the struggle, and even fear, that many students face when taking the MCA-II and how that apprehension often turns into low-test scores. Our ELLs become even more nervous and often score lower than their mainstream peers, particularly on the constructed response, or open-ended, portion of the exam. As a concerned ESL teacher, I am alarmed these low scores and by the frustration of my students.

After conferencing with students and listening to their questions in math class, I found that the biggest MCA-II problem area for ELLs was the word problems. Since the MCA-II requires that all students be able to solve word problems, I became interested in determining what the most effective strategy was for teaching word-problem solving. My school has decided to use the *Holt Middle School Math: Course 1* (Bennett, 2004) word problem-solving plan, adopted from Polya's strategy, to teach students how to interpret and answer word problems. Polya's strategy (1945, as cited in Bennett, 2004) encourages students to follow a series of steps, which ask students to understand the problem, make a plan, solve to find the answer and look back to check their work. Because my school is already using this plan, I became interested in evaluating it to see whether it is an effective strategy for ELLs. I began this investigation by researching what past mathematicians and linguists have said about ELLs and their struggles with math.

This chapter introduces the current theories of mathematics instruction, issues associated with the mathematics register, student struggles with word problems and mathematical accuracy, and recommendations for dealing with student struggles. It will

also describe the areas in which research is lacking on the topics of ELLs and their understanding of mathematical word problems.

The Theory of Teaching Mathematics

For years, mathematics teachers across the country taught in a traditional, teacher-centered format, giving lecture-based lessons where students were asked to do practice problems and memorize basic facts. The National Council of Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards (1989) changed traditional methods of mathematics teaching, regarding problem solving as a method of teaching that allowed students to think for themselves and be a part of the learning process. From that point on, math teachers began to incorporate problem solving strategies and activities, and started to stress the importance of student-centered thinking.

Because the teaching of math has changed drastically in the last twenty years, district standards have also become more problem-solving focused. Many questions on state tests have become word problems so that students have the opportunity to demonstrate their ability to think critically and problem solve. For many students, the standardized tests can be a valid measure of problem-solving skills and mathematical abilities. As Martiniello (2008) explains, the complication begins with ELLs who are trying to problem solve in a second language. The accuracy of standardized-test scores can become a problem “if the student is ELL and the math test includes questions that the student might have trouble understanding” because it is “unknown whether a low score is due to the student’s lack of mastery of the math content, limited English proficiency, or both” (Martiniello, 2008, p. 2). Since ELLs are required to take these standardized tests,

teachers try to determine the language issues that might arise when students attempt to problem solve in their second language.

The Language of Math

For many secondary students, the language of mathematics becomes difficult. Students come across hundreds of new vocabulary terms and grammatical structures that are unique to the field of mathematics. Halliday (1975, as cited in Cuevas, 1984) calls language with specialized meaning in mathematics the *mathematics register*, also termed the *discourse of mathematics* (Carr, et al., 2009). Students must learn both math-specific vocabulary and more common words that have a special mathematical meaning (Halliday, 1975 as cited in Cuevas, 1984). All students are expected to learn math-specific language or terms such as *quadratic*, *algebraic*, and *quotient*, in order to be successful in math (Carr, et al., 2009, p. 43).

The mathematics register also contains common words that overlap with other content disciplines. When ELLs are introduced to secondary mathematics, they are expected to become proficient in both the common words and math-specific vocabulary sometimes, while still learning basic conversational English.

When working on mathematical word problems, ELLs must also be aware of the style of language used to argue a point or support an answer. The type of language used when writing or speaking about conclusions is yet another complex area of the mathematics register that ELLs must overcome in order to be proficient mathematicians (Halliday, 1975 as cited in Cuevas, 1984).

Lager (2006) stresses the common misconception that mathematics is a universal language or that math requires little linguistic understanding because it is based on symbols and numbers. For ELLs, mathematics can be quite daunting because it is now language-focused with the addition of word problems and other listening, speaking, reading, and writing activities.

Student Issues with Word Problems

Often mathematics teachers find that students struggle most with word problems. As Cummins (1988, as cited in Johnson, 2010) explains, students have difficulty “recognizing the mathematical or cognitive demand, and understanding the problem in its context” (Johnson, 2010, p. 65). Students who lack the linguistic knowledge needed to comprehend the demands of the math problem have particular difficulty. A student is unable to solve a word problem without first knowing what the problem is asking him or her to find. Word problems that are cognitively demanding require students to interpret information they are asked to find and think critically about how they might find a mathematical answer. As Chamot, Dale, O'Malley & Spanos (1992) state, “learning to comprehend sufficiently well to interpret meanings embedded in the context of a word problem is now a math problem solving prerequisite.” Because of the high level of language required to comprehend word problems, ELLs must be taught clear comprehension strategies in order to successfully solve word problems.

Another issue that arises is the possibility that an ELL may not have had much schooling in his or her first language. ELLs coming to the United States may be refugees seeking asylum or migrant workers moving with the harvest. These students often lack

prior formal schooling and, therefore, struggle to read or write. They are rarely exposed to the language of math and are unable to understand word problems.

Recommendations for Student Struggles

While several researchers have recommended that secondary students be taught how to solve mathematical word problems through the teaching of higher-order thinking skills and pre-set problem solving plans, little of this research included ELL participants in their studies. Basurto (1999) states that reading-comprehension strategies are the most effective in teaching ELLs how to solve mathematical word problems. ELLs need to be able to predict, question, and determine meaning of vocabulary while solving word problems. Basurto (1999) claims that if math teachers are able to effectively incorporate reading strategies in math content, ELLs are more likely to be successful when solving word problems. However, her research does not reference studies where reading comprehension strategies were tested along with the solving of math word problems. As far as I can determine, Chamot, et al. (1992) were the only researchers to test the use of reading strategies with ELLs when they were solving word problems. More research could be done to clearly outline the effectiveness of reading comprehension strategies in improving math word problem-solving performance among ELLs.

Huang and Normandia (2007), in a study of high school students, explain that in order to prepare ELLs for the language-intensive work necessary to solve word problems, it is important that students find a way to connect social communication with academic writing. As Huang and Normandia state, mathematics instruction is most beneficial when the social language that students are comfortable with can be combined with written

tasks. Even ELLs are quickly able to participate in social activities, making them feel as though they are part of the class and that their opinions are valid. More research could be done to determine the effects of social communication and academic writing in mathematics with middle school students.

Zwiers (2008), in his studies of ELLs in secondary social studies, science, and language arts classes, states that think alouds and cooperative groupings where ELLs are given the opportunity to orally state their ideas while they are thinking and before they write are great ways to foster student success. More studies can be done to see whether think alouds and cooperative groupings are also effective in improving performance in middle school mathematics classrooms.

Fogelberg, et al. (2008), in their research with native English-speaking elementary students, describe the benefits of writing in mathematics, stating that students often produce more thoughtful answers and are able to communicate more higher-order thinking questions and responses. Fogelberg, et al. explain that often writing improves mathematical achievement when students are using it to answer word problems. When writing about word problems, students are able to explain their mathematical knowledge, strategic knowledge, and the process of their thinking (Fogelberg, et al., 2008). More research could be done to find out how secondary ELLs perform when asked to communicate their thoughts in writing.

Holt Middle School Math: Course 1 (Bennett, 2004), used by the middle school where I teach, supports a word problem-solving strategy (Polya, 1945 as cited in Cuevas, 1984) that requires students to complete four steps in order to solve word problems.

These steps are intended to provide structure for students. Step one asks the reader to understand what the problem is asking him or her to find. Step two asks the reader to make a plan to solve the problem. Step three asks the reader to solve the problem and step four requires the reader to look back at his or her work and check for errors. This strategy is similar to many other reading-comprehension strategies because it asks students to question and determine the meaning of the word problems they read. After reviewing the *Holt Middle School Math: Course 1* (Bennett, 2004), I began to wonder how effective Polya's problem-solving plan would be when used by ELLs. Only Chamot, et al. (1992) have completed studies on middle school-aged ELLs who use Polya's problem-solving steps in conjunction with reading comprehension strategies. Overall, it appears that many of the recommendations made in the above studies lack sufficient research regarding their use with ELLs. More research can be done to determine the effectiveness of Polya's plan as a singular strategy in improving ELL oral and written communication when responding to mathematical word problems.

Guiding Questions

My research question asks: How does Polya's problem-solving plan affect a sixth grade English Language Learner's ability to solve mathematical word problems and explain his or her thinking both orally and in writing? Supporting questions include: Do students use Polya's plan to solve word problems? How fluently are they using Polya's problem-solving steps? Over time, do students use more steps and use more detail when explaining what they did in each step?

Background of the Researcher

My participants were sixth graders in my co-taught advanced ELL math class. These students were taught by both a mathematics-licensed teacher and an ESL teacher in order to improve both language and mathematics skills. I completed all research and testing in my classroom during the regular school day. My participants were interviewed at lunchtime with parental consent. The word problems given to these students were part of the mainstream curriculum of our school and did not deviate from the school-wide mathematics pacing guide.

As I began my research, I realized that I needed to be aware of my own personal biases that may affect the validity and reliability of my study. Since all of my teaching experience has surrounded the reading and writing ability of ELLs, I knew that I had to be conscious of how my students' writing abilities compared to that of their native-speaking peers. I asked my co-teaching partner to review all ELL writing samples so as to avoid preferential treatment of ELLs. I was also aware of my familiarity with ELLs' writing and worked to avoid interpretation of writing that is incomplete. To avoid issues with favoritism, I tested sixth grade students whom I had not previously taught. Finally, I recognized that my personal feelings about the unrealistic and inaccurate expectations of the MCA-IIs may have affected my teaching and monitored my tone and attitudes when discussing portions of the MCA-II with students.

Summary

In this study I focus on the effectiveness of our school's problem-solving plan in teaching ELLs how to think about a word problem, how to structure their written answers

and how to communicate those answers accurately and efficiently. I will determine whether Polya's plan functions as a tool to help students gain confidence in their ability to solve word problems. Since ELLs struggle most with word problems, it becomes apparent that we, in the teaching community, need to investigate whether the strategies we use are the best practices for ELLs. If we do not address this issue, it is likely that we will continue to see low MCA-II test scores due to our students' inability to effectively communicate their mathematical understanding.

This research is meant to determine how effective Polya's problem-solving plan is in improving the quality of written and oral responses to word problems when ELLs approach math problems. It is my hope that this study will shed light on the needs of ELLs in middle school math classes so that math teachers are able to effectively instruct this group of learners.

As an ESL teacher, my colleagues look to me to provide strategies and best practices for teaching ELLs. Since I am not trained in the teaching of mathematics, it is sometimes difficult for me to give informed advice. In this study, I hope to learn more about how our problem-solving strategy is influencing ELLs and their ability to use oral and written language so that I am able to help my math colleagues when content-specific questions arise.

It is my responsibility as an ESL teacher to give ELLs the language skills they need to be successful in mathematics. It is my hope that this research will highlight key information on the effectiveness of Polya's problem-solving plan so that future teaching

will reflect the best interests ELL students. This research can be seen as the support for any future changes in word problem teaching practices.

In Chapter One, I related the purpose, significance, and need for this study. The context of the study was briefly introduced as was the role, assumptions and biases of the researcher. The background of the researcher was also provided. In Chapter Two, I provide a review of the literature relevant to mathematics problem solving, word problem solving strategies, the relationship between writing and thinking about word problems, and factors that affect student confidence in mathematical ability. Chapter Three includes a description of the research design and methodology that guides this study. Chapter Four presents the results of this study. In Chapter Five, I reflect on the data collected. I also discuss the limitations of the study, implications for further research, and recommendations for math teachers who have struggling ELLs in their classrooms.

CHAPTER TWO: LITERATURE REVIEW

English as a Second Language (ESL) teachers are quickly aware of the linguistic struggles that students face in all content areas. When ESL teachers review curriculum, mathematics is often overlooked due to false ideas about its linguistic complexity. After reviewing the research, I was led to focus on problem solving, and wonder how language instruction can be included in problem solving. Teachers often note that ELLs struggle most with the language of word problems. This study is meant to find out if Polya's word-problem strategy is effective in improving ELLs' ability to communicate their thinking both orally and in writing.

In this literature review, I will investigate recent theory about mathematics teaching; the linguistic features of mathematics; the issues students face when working with word problems; and the relationship between mathematical understanding, speaking and writing.

Recent Theories on Mathematics Instruction and Assessment

Focus on Problem Solving

Many teachers remember the days of drill and practice math when teachers performed lectured-based lessons and asked students to memorize and repeat basic facts or theorems. Lessons used to be concentrated on numbers and formulas, not language. Researchers have come to find that when many people recall their mathematics memories

and still believe that doing math requires few academic language skills such as reading or writing (Lager, 2006; Lass, 1988). Teachers from content areas such as social studies or science may feel that math is a universal language in which less reading and writing are needed for comprehension. In 1989, however, the National Council for Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards described the need for a change in mathematics instruction from the traditional concept-centered to a new language-centered problem-solving model. According to NCTM (1989), the Curriculum and Evaluation Standards for mathematics have changed to concentrate on problem solving in order to produce students who are better able to use language to communicate about their thinking. Several researchers have discussed how problem solving can be used to answer nearly all mathematical questions (Van de Walle, 2004; Bender, 2005). The switch to problem-solving-focused instruction occurred because it gives all learners a chance to begin solving math problems at the appropriate developmental level (Van de Walle, 2004). When teachers use differentiated critical thinking questions in problem-solving activities, Van de Walle states that students at every developmental level are able to expand the scope of their thinking. Problem solving also becomes part of the learning process and can be connected to real world problems (Van de Walle, 2004). Along with problem-solving strategies come metacognitive development, which leads to an improved ability to express one's thinking (VanSciver, 2008; Bender, 2005; Van de Walle, 2004). As Bender (2005) explains, metacognitive instruction, or the explicit teaching of critical thinking, allows for students to take responsibility for their mathematical thinking by monitoring their work and devising a plan of action for solving the problem. Researchers

have stressed the importance of explicitly teaching students how to work metacognitively, stating that students who are taught how to plan their own steps when solving problems are more likely to be successful (Bender, 2005; Schoenfeld, 1992, as cited in Van de Walle, 2004; VanSciver, 2008). While these researchers refer to the effectiveness of problem solving for all middle school learners, they do not explain how their problem-solving approach has specifically affected ELLs.

Standardized Testing

With the rise in high-stakes standardized testing, researchers have addressed the need for a focus on math questions that require problem-solving skills (Martiniello, 2008; Lager, 2006). Many states' mathematics tests contain word problems where students are expected to explain their thinking in writing (e.g., Wisconsin Department of Public Instruction, 2006; New York State Education Department, 2009; Michigan Department of Education, 2000). As was stated above, students need to have developed their metacognition and problem-solving skills in order to solve multi-step word problems with open-ended responses. Martiniello (2008) explains that the No Child Left Behind Act (NCLB) (2001) requires that all students take their state's standardized tests one year after entering the country. When test results are announced, ELLs are separated into their own group to show their progress. ELLs are consistently failing to meet state expectations, causing wide frustration and confusion among teachers, students, and administrators.

In her 2008 study, Martiniello studied ELL fourth graders as they took the Massachusetts Comprehensive Assessment System to determine how ELLs performed

when confronted with linguistically complex math word problems. She found that longer, linguistically complex problems caused the most difficulty for ELLs. Lager (2006) also analyzed the language of math word problems in interviews with Spanish-speaking ELL sixth and eighth graders. He found that ELLs made the most errors in linguistically challenging problems. VanSciver (2008) describes the struggles that native English-speaking middle and high school students have with the reading aspect of math word problems, explaining that standardized tests are as much of a reading-ability test as a math test. These researchers have found that there is no way to create a standardized math test that does not also test the linguistic ability of students (Martiniello, 2008; VanSciver, 2008; Lager, 2006). While these researchers did investigate how ELLs and native English speakers performed on mathematical word problems, methods for improving ELL understanding of linguistically complex word problems were not addressed in their studies.

Several researchers have studied the linguistic complexities of mathematics for both ELLs and native-English speakers. These studies will be discussed in the following section.

The Language of Math

As the math standards and standardized tests continue to become more problem solving focused, the demand for language use and understanding increases. If students are to become higher-level thinkers who are able to show mathematical comprehension, they will be expected to demonstrate their understanding through more complex language (VanSciver, 2008). As Cuevas (1984) states, mathematical understanding is determined

by both the metacognitive process and the ability to use language to explain that thinking. In his 1981 study (as cited in Cuevas, 1984) of ELLs, Cuevas explains his Second Language Approach to Mathematics Skills (SLAMS), which is a series of math and languages strategies meant to improve ELL performance on math word problems. His work with secondary ELLs led to the development of these two strands of mathematics teaching. According to these researchers, the teaching of language is an essential part of teaching math.

Linguistic Factors

Many researchers have described the complexity of language in mathematics by describing the mathematics register, which includes both common academic language that has a specialized meaning in mathematics and math-specific vocabulary (Halliday, 1978 as cited in Schleppegrell, 2007; Cuevas, 1984; Carr, et al., 2009). A mathematics register is “a set of meanings that function in a particular way and belong to the language of mathematics” (Halliday, 1978, p. 195 as cited in Schleppegrell, 2007). When students begin their schooling, they describe things using a more informal, everyday language. As they gain more knowledge and more academic language, they move from informal to formal explanations of complex concepts (Schleppegrell, 2007). For ELLs, this process may take longer since they are describing concepts in their second language. A mathematics register includes words or phrases that are specific to the context of mathematics. Schleppegrell (2007) explains that the distinction between mathematics language and other content languages helps remind us that we construct mathematical knowledge in a different way. Researchers have uncovered several linguistic features

that are used in mathematics in the areas of vocabulary and grammar (Cuevas, 1984; Schleppegrell, 2007, Carr, et al., 2009; Zwiers, 2008; Johnson, 2010; Lass, 1988; Martiniello, 2008).

Vocabulary. Vocabulary is one aspect of language that is quite complicated for ELLs, especially when they encounter math-specific content words. All students are expected to learn words such as *quadratic*, *algebraic*, and *quotient*, for example, in order to be successful in math (Carr, et al., 2009). Some of the words used in mathematics are common in everyday language but do not have the same meaning (Johnson, 2010). For example, *mean* in mathematics refers to an average of a set of numbers, but in everyday English the meaning changes to the significance of a word or an act of cruelty. Other words, such as *quotient* or *quadratic* (Carr, et al., 2009, p. 136), are used only in math. Phrases, or locutions, such as *least common multiple* (Cuevas, 1984, p. 136) fit into the mathematics register because they are worded in a way that makes them specific to math. ELLs may be familiar with the everyday language meanings of compound words such as *output*, but again, these words may have a distinct meaning in math. English Language Learners may be thrown off by homophones as well (Cummins, 1981 as cited in Johnson, 2010). Homophones such as *cents* and *sense* (Johnson, 2010, p. 101) are confusing to ELLs because they sound similar but have different meanings. Words that sound alike such as *fourteen* and *forty* may also cause trouble for ELLs.

Grammar. Grammar structures are part of the mathematics register as well. Mathematical phrases are written with a certain style and only function within that style (Cuevas, 1984). Phrases such as *the area under the given curve* (Cuevas, 1984, p. 136)

are written in a way that is unique to the field of mathematics. These phrases and structures have to be explicitly taught in order for student recall to take place. Dawe (1983) observed bilingual middle school students in England as they worked through math word problems and found that they struggled most with logical connectors such as *in order to* or *because*, which are used in math to transition from one thought to the next. ELLs may be unfamiliar with these types of connection words so teachers will have to explicitly teach these phrases. In math, modifiers are often used to change the meaning of a word (Dawe, 1983). This becomes difficult for ELLs when they attempt to distinguish between a *pentagon* and a *regular pentagon*.

Other Factors

Often times, ELLs are confused when entering U.S. mathematics classrooms because they are accustomed to different algorithms or steps used to solve problems. For example, division, multiplication, addition, and subtraction problems are written differently in many countries. Commas, periods, and geometric proof procedures are also used differently in other countries (Lass, 1988; Johnson, 2010; Zwiers, 2008; Cuevas, 1984). A student coming from Mexico, for instance, might write the number *1,000* as *1.000*, replacing the comma with a decimal point. These small differences change the meaning of the numbers in a major way.

When confronted with the many linguistic features of mathematics, it becomes apparent that language comprehension is a key piece of mathematical comprehension. As math teachers build problem-solving skills, they should focus on language skills (Basurto, 1999).

Mathematics teachers often voice a concern over the struggles their students have when making sense of word problems. As we have seen, language plays a large part in student battles with word problems. Students may also become frustrated with the many different types of word problems they face in math class. Cummins (2006) describes these different types of word problems on a continuum from cognitively undemanding to cognitively demanding and from context-reduced to context-embedded. Context-reduced word problems are difficult for ELLs because they do not include any linguistic or visual clues that might help students connect with the scenario being presented. Context-embedded problems are much more likely to include hints or triggers that might allow ELLs to connect to background knowledge or make the content more comprehensible. When any student encounters a word problem, one step is to determine how he or she will solve the problem. If the problem is cognitively undemanding, it is likely to have one or two small basic steps, and the student probably has the problem-solving skills necessary for solving it. If the problem is cognitively demanding, however, it is much less likely that the student will be able to figure out how to solve the problem, especially if it is context reduced. When mainstream students are presented with word problems that require them to think critically but are not given a strategy to make the content comprehensible, it is unlikely they will have success when trying to solve the problem (Johnson, 2010). When ELLs attempt the same problem, it is even less likely that they will be able to solve because they may be struggling with the language of the problem as well as the mathematic elements.

Along with linguistic struggles come issues related to speaking and writing. A summary of the research done on oral and written communication in mathematics will be explained in the next section.

Relating Speaking and Writing to Mathematics

Writing and Math

As was discussed in the Recent Theories on Mathematics Instruction and Assessment section of this chapter, metacognitive processing is a crucial skill for students to possess in order to effectively solve word problems. Writing is closely linked to metacognition, according to a study of native English-speaking high schoolers (Huang and Normandia, 2007), because it is a vehicle for students to express thought processes. In their study, Huang and Normandia (2007) interviewed students and reviewed writing samples and found that the types of linguistic structures students used in written responses to word problems were directly linked to mathematical accuracy. The study also showed how oral and written language are inextricably linked to mathematical understanding, making it necessary for language strategies to be taught along with mathematical concepts. Wilde (1991) found that when native English-speaking first graders were taught the language strategy of writing their ideas down before explaining their thoughts about math problems, they were able to include more details and felt more successful. She recommended using writing every day in math because it gives a student the opportunity to express new information in his or her own words, which increases knowledge and understanding. She claimed that mathematical writing leads to expository writing because students are able to not only connect to the new knowledge they have

gained but also write about it from their own perspective. These aspects of writing give way to deeper comprehension and allow teachers the opportunity to assess the students' level of understanding as well. When students write expressively and informally about their understanding of various math concepts, they become less concerned with presentation of their thoughts and the process then becomes more like a think-aloud on paper (Wilde, 1991).

Researchers explain that having students write their own word problems is another way that students are able to see the connection between the verbal and symbolic expression of math (Wilde, 1991, Fogelberg, et al., 2008; Van de Walle, 2004; Smith, Miller & Grossman, 1992). Murray (2004 as cited in Fogelberg, et al., 2008) describes one of the benefits of writing in math to be the ability of writing to naturally encourage revision and questioning of written information. When a student is puzzled or unsure of a solution or step involved in problem solving, writing allows the student to recheck thinking and discover new insight or error (Van de Walle, 2004).

In a long-term study of native English-speaking college students in mathematics courses, Smith, Miller & Grossman (1992) found that teachers who taught reading and writing strategies explicitly had students who could clearly explain their mathematical thinking in writing. Students who were not given this direct instruction struggled to write about their thoughts. This study demonstrates how writing and reading strategy instruction can be included in mathematics curriculum. While studies about the importance of writing instruction have been done with secondary native English speakers,

little research outlines the specific benefits of this type of instruction for middle school ELLs.

Speaking and Math

Writing and speaking are often linked when communication skills are discussed in mathematics. For ELLs, oral communication is important because these students may not know how to properly structure writing but may still be able to communicate their thinking orally. Oral communication allows ELLs to be a part of problem-solving discussions. Huang, Normandia, and Greer (2005) studied the oral discourse of native-English-speaking high schoolers as they explained their thinking in math class. These researchers collected data through classroom observation, audio recordings, interviews, and student work. They found that students are more likely to succeed in explaining their thinking aloud when they are taught how to use a think-aloud strategy. Students need to practice oral communication in order to learn mathematics and in order to understand how to speak mathematically (Huang, Normandia & Greer, 2005). As students think about mathematics, they try to organize their thinking so they are able to communicate. It benefits mathematical understanding more if students are given activities that allow them to converse with their peers or teachers as they try to convey their thoughts (Huang, Normandia & Greer, 2005). Tan (1999) also found that instruction in how students should orally explain problem-solving steps is one of the best ways for ELLs to learn and recall for future math problems. In the above studies, both ELLs and non-ELLs were observed as they completed mathematical word problems. These studies investigate the connection between writing, oral explanation, and mathematical accuracy when ELLs

solve word problems. None of the studies explain the best strategies for teaching students how to think aloud nor do they mention how instruction in a step-by-step problem-solving plan might affect ELLs ability to explain their thinking orally or in writing.

Polya's Problem-Solving Plan

Since there is a clear need for explicit teaching of problem-solving strategies, it is important that schools include this type of teaching in their curriculum (Cuevas, 1984). In 1945, Polya described a four step word-problem-solving strategy that is based on best practices in reading comprehension. This four-step strategy is found in many mathematics textbooks, including *Holt Middle School Math: Course 1* (Bennett, 2004). The plan also contains more specific substeps, which give students more guidance as to how to use each step (Table 2.1). Step one of the strategy asks the reader to understand the problem by reading the problem and explaining what question is being asked. Next, the reader identifies what information is already given. The second step requires the reader to develop a plan for solving the problem. After the reader has written all steps, asks the reader to solve the problem and perform all necessary calculations. The fourth step asks the reader to look back at the work and check to make sure there are no errors (Polya, 1945).

In order to discover effective problem-solving strategies for ELLs, Chamot, et al. (1992) collected audio interview recordings and student responses to math word problems in their studies of elementary and secondary ELLs who were explicitly taught Polya's (1945) problem-solving strategy along with other Cognitive Academic Language

Learning Approach (CALLA) instructional strategies. These researchers discovered that ELLs who received explicit instruction in math problem-solving strategies were better able to answer problems with more mathematical accuracy. Their results showed a significant difference in mathematical accuracy and communication skills among students with higher math ability, especially when those students were instructed to use the problem-solving strategies.

Table 2.1

Polya's Problem-Solving Plan

Step One: Read and Understand the Problem	Substep 1: Tell what the question is asking you to find Substep 2: List what information you already have
Step Two: Make a Plan	Substep 1: List all math steps you will need to take to solve
Step Three: Solve the Problem	Substep 1: Do the math
Step Four: Look Back and Check	Substep 1: Look back over your work for any errors Substep 2: Use a strategy to check your work

Schema-Based Instruction

Other researchers also state the importance of a problem solving plan similar to Polya's (1945) plan but include the addition of schema-based instruction. SBI is instruction where students are taught to identify the common patterns or structures present in a problem as opposed to general-strategy instruction (GSI), where students are taught to use a general problem-solving plan. Griffin and Jitendra (2009) studied random groupings of sixty third graders to determine whether SBI or GSI was more effective in

improving the students' ability to solve mathematical word problems. According to Griffin and Jitendra (2009), students who receive SBI, or study the common structure or pattern present in particular word-problem contexts, were more likely to accurately answer word problems. Students who were taught a straightforward problem-solving plan similar to Polya's (1945) also improved their ability to solve word problems accurately but did not experience as much gain as those who were taught with SBI. Xin (2008) also studied the effects of SBI on learning disabled (LD) fifth graders. In his study, the fifth graders were introduced to SBI and taught to use a problem-solving plan similar to Polya's plan. He found that these students were able to increase their mathematical skills as a result of their SBI-explicit instruction. Several researchers explain that SBI was more effective in improving mathematical accuracy among elementary students (Griffin & Jitendra 2009; Xin, 2008) but do not mention the effect a strategy like SBI might have on improving a student's ability to communicate his or her thinking. These studies similarly do not mention the affects SBI might have on ELLs.

Strategies Used by Proficient Problem Solvers

A form of Polya's problem-solving plan, called the "Multidimensional Problem-Solving Framework" (Carlson & Bloom, 2005, pg. 45) was used in a study where college-level mathematicians and math education majors were examined to find out how they worked and thought as they solved word problems (Carlson & Bloom, 2005; Cifarelli & Cai, 2005). In these studies, researchers were interested in focusing on the factors that make a person an effective problem solver. Cognition and metacognition were found to be two key features of successful problem solving. Both groups of

advanced mathematicians excelled when asked to communicate all parts of their thinking process while solving math word problems. They were able to describe all of the methods that they could use to solve each problem and why they chose the methods they ended up using to find their answers. When they made mistakes, they were able to self-correct and explain why they believed they made those mistakes. These studies could be used to determine a method teachers might use in training students in what good problem solvers do when encountering math word problems. While this research did concentrate on communicating about thinking while solving word problems, it was strictly based on data collected from high-level mathematicians who were native English speakers and did not divulge information that could easily be applied to middle school mathematics teaching.

Ishida (2002) researched sixth grade Japanese students who had been instructed in several problem-solving strategies beginning in second grade to determine how they communicated about their thinking when solving word problems. Because these students had a great deal of background in the use of effective problem solving strategies, they were able to solve word problems with accuracy but lacked the metacognitive understanding of those strategies needed to clearly explain why they chose various methods when solving problems. The information collected in this study describes how middle school students can communicate their mathematical thinking but focuses only on native-speaking students with years of prior strategy knowledge and practice.

Other researchers have focused non-native English speakers who have had success with math problem-solving when word problems were taught in combination

with language and reading strategies such as prediction and questioning are quite effective in promoting problem-solving (Basurto, 1999). Basurto (1999) interviewed two bilingual third and fourth grade teachers and one two-way immersion first grade teacher to determine how instruction in reading comprehension strategies can help native Spanish-speaking students solve mathematical word problems. In her study, she found that ESL teachers need to include opportunities for ELLs to connect with their own experiences and build background with math word problems in order to increase mathematical understanding.

The above research explains the effectiveness of instruction in reading comprehension strategies as a way of developing problem-solving skills in elementary ELLs (Chamot, et al., 1992; Basurto, 1999). It also describes the word problem-solving success that secondary ELLs have when they are taught how to use Polya's (1945) problem-solving plan. Some of the research explains the crucial role that communication about thinking may play in becoming an effective problem solver. These studies have not addressed how explicit instruction in Polya's problem-solving plan might directly affect middle school ELLs' ability to explain their thinking orally and in writing.

The Gap

As this chapter demonstrates, there are many factors that affect an ELL's ability to solve and interpret word problems. The issues ELLs face become even more complex as they come to middle school where the math problems become more abstract, the language becomes more complex, and the social pressure to fit in intensifies.

Some studies have concentrated on the importance of teaching problem-solving based-mathematics at the middle school level. Most of these studies have included data from native English speakers but have not focused on accommodations that might need to be made for ELLs. Other studies addressed ELLs and the teaching of mathematical problem solving but did not disclose effective methods for making linguistically complex text more comprehensible for ELLs. The studies clearly show that writing and reading strategy instruction must be included in a mathematics curriculum. While studies about the importance of writing instruction have been done with secondary native English speakers, little research outlines the specific benefits of this type of instruction for middle school ELLs. Studies have also shown that successful problem solvers are able to communicate about their thinking but little research has been done to show effective ways teachers might help middle school students with little prior problem-solving background to improve their communication or metacognition when solving math problems. While Chamot, et al. (1992) describe the word problem-solving success that secondary ELLs have when they are taught how to use Polya's (1945) problem-solving plan with other strategies, little research has been done to show how explicit instruction in Polya's problem-solving plan might directly affect middle school ELLs' ability to explain their thinking orally and in writing. A change in ELLs' ability to explain their mathematical thinking both orally and in writing has not been documented.

Research Questions

My research question asks: How does Polya's problem solving plan affect a sixth grade English Language Learner's ability to solve mathematical word problems and

explain his or her thinking both orally and in writing? Supporting questions include: Do students use Polya's plan to solve word problems? How fluently are they using Polya's problem-solving steps? Over time, do students use more steps and use more detail when explaining what they did in each step?

Summary

In this chapter I have highlighted key research done on mathematics teaching theory; linguistic features of math; common student struggles with word problems; and research on the connection between math, writing, and speaking.

In Chapter Three, I will describe the methods, instruments and procedures I will use in my study of middle school-aged ELL students in a math classroom.

CHAPTER THREE: METHODOLOGY

This study was designed to explore how Polya's problem-solving strategy (1945) affected English Language Learners' (ELLs) ability to communicate thinking, with both oral and written fluency, when approaching word problems. In this study, I wanted to know if Polya's plan served as an effective tool for ELLs when trying to structure their written responses to word problems or when trying to communicate their thinking. In this chapter, I will explain how I implemented and tested Polya's problem-solving strategy in my middle school classroom. I will describe the type of research I used, who my participants were, the series of word problem assessments I created, and the methods I used to collect my data.

To answer my question, I needed to collect data from recordings of retrospective verbal reports and examples of students' written responses to word problems.

Chapter Overview

This chapter describes the methodologies used in this study. First, the rationale and description of the research design are presented along with a description of the mixed method research paradigm. Second, the data collection protocols are presented. Next, I will present background information on my participants and school population. I will continue with a discussion of how I analyzed my data and finish with a description of what I needed in order to efficiently collect my data.

Mixed Method Research Paradigm

Qualitative Research

My research question concentrated on a small portion of middle school ELLs and their mathematical thoughts and communications. As Merriam states, “the overall purposes of qualitative research are to achieve an understanding of how people make sense out of their lives, delineate the process of meaning-making, and describe how people interpret what they experience ”(Merriam, 2009, p. 14). I realized that my topic and goals for this research were targeted at finding out which processes ELLs are using to think about word problems, so qualitative research seemed to be the most appropriate method.

Qualitative research encourages the researcher to be the main instrument for data collection and analysis (Merriam, 2009) and stresses the importance of questioning and observation. Since I was most interested in recording my students’ thinking through verbal reporting, qualitative research appeared to be the best practice for this kind of data collection. Qualitative research does not prove a hypothesis or come to a clear conclusion, but rather determines the status of a particular process or idea as it exists in its natural environment (Merriam, 2009). One type of qualitative data analysis is a priori classification. One way a researcher uses a priori classification (Merriam, 2009) is by using pre-existing categories to classify aspects of the data. Because I planned to describe the verbal report results and categorize them using a constant comparative method to determine similarities and differences, it appeared that a qualitative paradigm fit best with my study.

I also planned to use sets of numerical data, to determine the relationship between MCA-II word problem scores and checklist scores for Polya's problem-solving plan. Quantitative data collection would work best in both of these aspects of my study, thus this was mixed-method research.

Another important part of quantitative research is that data collection must be reliable and valid (McKay, 2006). In my study, I had my co-teaching partner score student writing samples using the same rubric as I did to ensure inter rater reliability and consistency. My co-teacher and I practiced grading with the rubric while collecting pilot study data in order to ensure consistency as well.

Data Collection

Description of Participants

In qualitative research, participants are chosen to reflect a purposeful sample of the natural environment during the study. As Merriam (2009) states, purposeful sampling is selected because of its ability to provide information-rich data about the event or process that the researcher is observing. In my research, I chose six sixth grade ELL students: five Latino and one Oromo. Of the six students chosen, four were female and two were male. I chose my participants from my collaboration math classroom because they were exposed to the same curriculum as their native English-speaking peers. I also chose my participants because they represent the largest populations of students who are being placed into mainstream content and elective classes. In quantitative research, my group of participants represented a sample of convenience chosen based on the needs of my study. I had complete access to these participants since they were my students. My

hope was that these participants would be able to provide data that might be useful for content teachers who have early advanced students in their classes.

Location/Setting

My research took place in a first ring suburban middle school near a large metropolitan area in the upper Midwest. There are approximately 600 students in the school. The school consists of a diverse population of learners including 30% Caucasian, 20% Latino, 4% African and 46% African American. Seventy three percent of the students are on free and reduced lunch. Twenty-three percent of the school population is made up of ELLs, most of whom speak Spanish or Somali.

Pilot Study

I completed a pilot study to ensure that both my exam rubric and interview recorder functioned properly during my data collection. I administered the MCA-II practice questions to my sixth grade math class as a part of the regular instruction time. I asked the students to think about the problem-solving strategy we talked about in class and told them to show all their work. Once the students completed the exam questions, I sat down with my co-teaching partner and we discussed what each student should receive according to both Polya's Problem-Solving rubric (Table 3.1) and the MCA-II rubric (Table 3.2). This ensured grader reliability.

I also asked my participants to come in during lunch to talk about and record their thoughts on a word problem. I explained that they should tell me everything that they did to solve the given word problem and that I was going to record their response on my computer but would show no one else what they said. I let them know that I would

record them three times that year and that it should only take ten to fifteen minutes. I was sure to record students in my classroom when no other students or teachers were present and when there was no other sound so that I could get the best recordings possible.

Table 3.1

Polya's Problem-Solving Plan Checklist

STEP	ITEMS NEEDED	COMPLETE?
Step 1: Understand the Problem	a. Tell what the question is asking you to find b. List what information you already have	a. _____ b. _____
Step 2: Make a Plan	a. List all math steps you will need to take to solve	a. _____
Step 3: Solve the Problem	a. Do the math	a. _____
Step 4: Look Back and Check	a. Look back over your work for any errors b. Use a strategy to check your work	a. _____ b. _____
	SCORE:	_____ / 6

Procedure

My data were collected over the course of the 2009-2010 school year. In past years, our middle school had agreed to use Polya's strategy as a cross-curricular problem-solving plan. The school was scheduled to continue its use in the 2009-2010 school year. In order to collect information on Polya's problem-solving strategy, I introduced the steps of the problem-solving plan at the beginning of the school year. My co-teaching partner

and I continued to use Polya's plan as our main strategy for teaching our students how to solve mathematical word problems.

Table 3.2

MCA-II Sample Rubric

SCORE	DESCRIPTION
4	Response contains correct solutions for: <ul style="list-style-type: none"> • 2 different possible ways you could purchase the 56 juice boxes • what would be their total costs • the price per juice box The responses adequately show or explain all work.
3	Correctly determines the answer to at least three questions with adequate supporting work. OR Correctly determines the answer to all questions with incomplete supporting work.
2	Correctly determines the answer to at least two questions with adequate supporting work. OR Correctly answers at least three questions with incomplete or missing work.
1	Some work relevant to the problem.
0	Response is incorrect or irrelevant.

Materials

To answer my research question, I needed to collect data from recordings of retrospective verbal reports and examples of students' written responses to word problems. To collect useful information regarding written responses to word problems, I created a word problem exam that required written responses to word problems. These

word problems were taken from the MCA-II sample questions and related to the topics we were currently studying in our classroom. I used a six-point rubric, which included all key points that students should provide in their responses. The rubric works in coordination with the checklist of Polya's steps and is described in further detail later in this chapter. In order to understand student thinking, I recorded students as they thought out loud about their written responses to word problems. I created a coding system to organize and analyze the recorded data.

Data Collection Technique 1

To gather data about my participants' ability to explain their mathematical thinking in writing, I gave a series of word problem exams spread throughout the school year. I took exam questions from the MCA-II fifth and sixth grade sampler (See Table 3.3) (MDE, 2008) so that students were able to practice solving word problems similar to those found on the state test. The exam questions corresponded to the curriculum topic we were currently studying in math class. Each testing period consisted of two exam questions. To score the writing, I used the MCA-II rubric (See Table 3.2) that corresponds with the word problem sampler questions. The rubric consisted of a five-point scale, ranging from 0, which is incorrect and irrelevant work, to 4, which is correct with all work shown.

I also scored writing samples on their use of Polya's problem-solving steps. Polya's problem-solving steps include: understand the problem, make a plan, solve, and look back and check your work. See Table 3.1 to find specific information needed for each step of the problem-solving plan.

Table 3.3

MCA-II Sample Constructed Response Question

Be sure to show all your work in your answer book.

20. Juice boxes are packaged two different ways.

- A package of 24 boxes costs \$12.98.
- A package of 4 boxes costs \$2.59.

You need 56 juice boxes.

Part A Find 2 different possible ways you could purchase the 56 juice boxes and the total cost for each way. Show or explain your work.

Part B Using your answer in part A, what is the price per juice box if you purchase the lower priced combination? Show or explain your work.

Data Collection Technique 2

To collect data regarding my participants' ability to explain their thinking orally, I conducted and recorded three retrospective verbal report interviews throughout the course of the school year (Table 3.4). As McKay (2006) states, verbal reports are one of the few available methods of finding out more about the cognitive processes, which go on when students are solving problems. I introduced Polya's problem solving plan by orally describing the four main steps of the plan to the whole class. I used the think aloud strategy to model how I expected them to think through each of the problem solving steps so that they had a clear understanding of how the strategy works. I presented these steps every time I assigned a word problem to the class and asked students to gradually work on explaining their thoughts orally as they worked through their word problems. My hope was that students would feel comfortable with the think aloud process before I began my interviews. The interviews were done individually rather than in a whole

group, so their peers did not influence the students' answers. In these interviews, the students were given another MCA-II word problem from the item sampler and asked to read the problem. After students read the problem, I asked them to solve the problem using any plan or strategy they wanted to use. As they solved, I instructed them to think aloud so I was able to know what they are thinking. Before students began their explanation, I asked them to write down their thoughts for each step of the problem solving strategy. Table 3.4 shows the schedule I used to collect recorded and written data from my participants.

Table 3.4

Data Collection Schedule

		January	March	May
Technique 1	MCA exam questions	2 exam questions-fractions	2 exam questions-geometry	2 exam questions-probability
Technique 2	Retrospective report	2 word problems-fractions	2 word problems-geometry	2 word problems-mean, median, mode, range

Data Collection Technique 3

In order to determine the change in participant fluency, I counted the number of pauses in each of the three rounds of the recorded retrospective reports. As Ellis and Barkhuisen (2005) explain, the number of pauses in a given speech sample gives the listener an indication of how long it takes the speaker to plan out their message before

they begin speaking. This method is a valid way of determining the general fluency of the speaker. In this study, a pause was counted if there was silence during the participant's explanation that lasted three or more seconds.

Data Analysis

Data Analysis for Technique 1

As I stated earlier, I compiled written samples of word problem answers from each of my five participants. Next, I scored their responses using the MCA-II sampler rubric as seen in table 3.2. I also used the Polya problem-solving checklist, which I created from the *Holt Middle School Mathematics: Course 1* (Bennett, 2004) problem solving strategy. Once I scored all exams with both the rubric and the Polya problem-solving checklist, I compared the data from the beginning, middle and end of the year. Each writing sample was given a score from zero to four based on the amount of mathematical work they showed and the accuracy of their computation. The samples were given a score on their ability to use Polya's problem-solving strategy. If I could check off all items on the checklist (Table 3.1), the writing sample received a six. If students failed to use all of the problem solving steps, they received an overall score of zero.

To analyze this data, I compared the beginning, middle and end MCA-II rubric scores with their beginning, middle and end problem solving checklist scores. To do this, I made a spreadsheet of each student's MCA-II rubric scores with a time of year designation (beginning, middle, or end). I also included each student's Polya checklist score on the spreadsheet. Once I included all data in my spreadsheet, I used Microsoft

Excel to make a bar graph of each student's data points. I created one bar graph for the students' Polya rubric scores and another for the students' MCA-II rubric scores. Each student's scores is represented by a different colored bar. Next, I looked at each of the student's bars to determine whether there is an observable relationship between the use of Polya's strategy and the MCA-II rubric scores. I will describe this relationship for each student in my results section. After creating my graph, I discussed the relationship of the two variables with my co-teaching partner to determine what type of mathematical relationship is represented. I will describe this relationship in great detail in the results section of this paper.

Data Analysis for Technique 2

After I collected three recordings from each participant, I transcribed the recordings and coded the thoughts into groups based on common themes and use of Polya's problem solving plan. As McKay (2006) states, a limited coding system is used when the researcher is trying to categorize specific events or processes occurring in his or her classroom. Since I focused on the use of Polya's strategy for problem solving, I used a limited coding system developed from the problem-solving steps. My limited coding system analyzed the data into groups according to the mention of each step in Polya's problem solving plan. The main groups included: *understands the problem, makes a plan, solves, and looks back and checks*. I used sub-groups in order to categorize details within each of the main groups. The sub-groups included *explains meaning of question, identifies needed information, identifies math strategies needed, explains what steps will look like, explains how to solve, identifies errors and uses a strategy to check answers*.

When working to code the students' answers, I designated a different color to each student. I highlighted all major phrase units in the student's designated color. Next, I cut each of the major phrases out and placed them in the appropriate group or sub-group. Many researchers refer to this method of analysis as cut and paste and recognize it as an effective way to sort qualitative data (Merriam, 2009). Once I finished grouping all comments, I described the strengths and weaknesses of the data according to what I saw.

Data Analysis for Technique 3

After I counted the pauses, I created a table to record the number of pauses for rounds one, two, and three of the collection. I compared the number of pauses between each of the rounds and determined whether the participants increased or decreased the number of pauses taken as time passed. A decrease in the number of pauses shows that the participants have higher fluency levels.

Triangulation

In order to ensure grader reliability, I asked my co-teaching partner to score the word problems with the same rubrics and determined whether there was inter rater reliability. I also collected data from verbal reports, written MCA-II samples and confidence questionnaires. By collecting several forms of data, I ensured validity and reliability.

Ethics

This study employed the following safeguards to protect participants' rights:

1. Research questions were shared with participants prior to any data collection.
2. Permission forms were obtained from parents of participants.

3. The Hamline University Human Subjects Review and all Hamline forms were submitted before any data was collected.
4. Names of participants and schools remained anonymous to protect reputations.
5. All transcriptions were literal and taken word for word in order to properly represent the participants of this study.

Conclusion

In this chapter, I described the methods I used to determine whether Polya's math problem-solving plan is effective in improving the fluency of written and oral communication skills of ELLs. I collected data in the form of verbal reports and written word problem samples. The next chapter presents the results of this study.

CHAPTER FOUR: RESULTS

This study took place in an English as a Second Language (ESL) collaboration mathematics classroom and during student lunch times in an ESL classroom. The study was completed in a small, first-ring suburban district in the upper Midwest with sixth-grade ESL students with intermediate and advanced language proficiency. I collected my data through a series of MCA-II Mathematics constructed response practice problems and retrospective reports, which were recorded as students explained their mathematical thinking out loud. Through the analysis of this data, I hoped to discover how Polya's mathematical problem-solving plan affected the students' fluency and accuracy when explaining their thinking aloud and in writing while solving math word problems.

MCA-II Practice Problems

Participants in this study were given two sixth grade level constructed response MCA-II practice problems from the Minnesota Department of Education website and other MCA test preparation materials during each of the three data collections (see Appendix A). All students in my sixth grade collaboration math class were asked to answer these questions during class and were given several examples of how to set up a word problem using Polya's problem-solving plan. I modeled each of the steps in the plan and performed think-alouds every time I explained word problem examples to the class. After students completed the MCA-II constructed response problems, my

collaboration partner and I used the MCA-II constructed response rubric and the checklist of Polya's problem solving steps to determine their MCA-II accuracy score and their Polya score for each problem (see Tables 3.1 and 3.2). The two MCA-II scores were averaged and are shown in Figure 4.1. The Polya checklist scores were also averaged and can be found in Figure 4.2. Only the data from the seven participants in my study were used in the table. Figure 4.1 shows three clear patterns in the results from the MCA-II scores. The participants whose scores made no improvement from the January data collection to the March data collection make up one pattern. Ariana, Melissa and Janette's¹ scores fit into this pattern. The participants who improved their scores each time they were tested make up another pattern. Eagle and Apple fit into this group. The last pattern is made up of those students who performed worse on their March problems than on their January problems. Maria and Esmeralda fit into this group. Because the data show three separate patterns, the data are inconclusive as to whether instruction in Polya's problem-solving plan improves mathematical accuracy.

The data from March to May show that all but one of the seven students improved their MCA-II rubric score throughout the collection period. Janette was the only participant whose MCA-II score did not increase from March to May. Her score stayed constant throughout the course of the study. All other participants showed growth in their rubric scores from March to May. Ariana, Melissa, Eagle, and Apple improved their score by 0.5 from March to May. The other participants, Maria and Esmeralda,

¹ Pseudonyms were used for all participants.

improved their scores by 2 from March to May. These data could be the result of a student's strength in one mathematical topic and weakness in another.

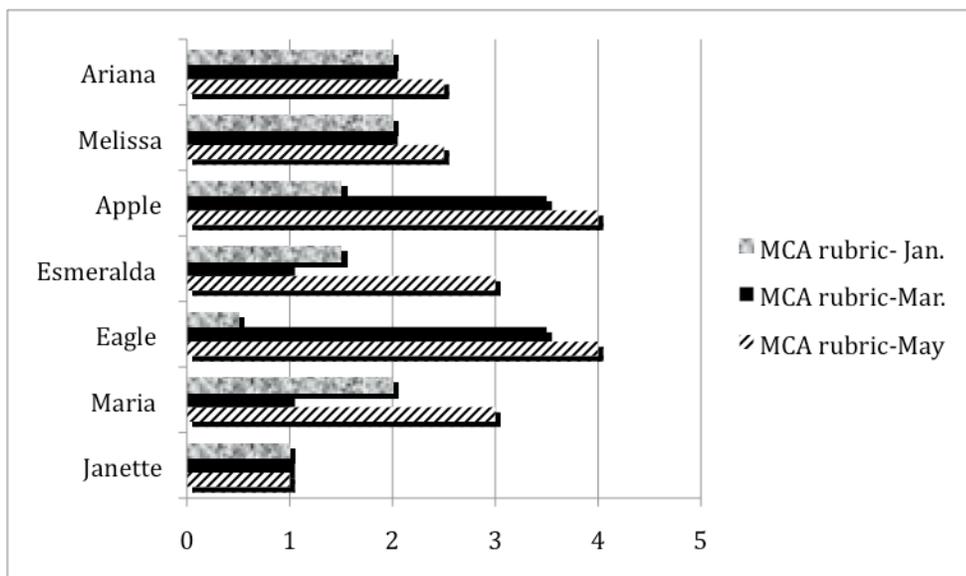


Figure 4.1. MCA-II Constructed Response Average Scores

Figure 4.2 shows that all participants' average scores from the Polya problem-solving checklist improved from January to May. All but one of the participants was able to increase the number of Polya's problem-solving steps they used from January to March as well. Only one student's average score stayed the same from January to March. From the results shown in Figure 4.2, it is clear to see that all participants increased their use of Polya's problem solving plan when writing down their mathematical steps for each MCA-II constructed response question.

Retrospective Reports

In order to determine whether the use of Polya's problem-solving plan improved the participants' ability to orally communicate their mathematical thinking, I recorded the students as they solved math word problems and explained their thinking aloud. After

the student data were transcribed, the participant statements were classified according to the categories listed in Table 4.1. The same method was used for recording rounds two and three. For the purposes of presentation, the transcribed conversations have been included in Appendix D and step numbers were inserted when a student used one of Polya's problem-solving steps. In Table 4.1, one can see how the step numbers relate to the Polya steps.

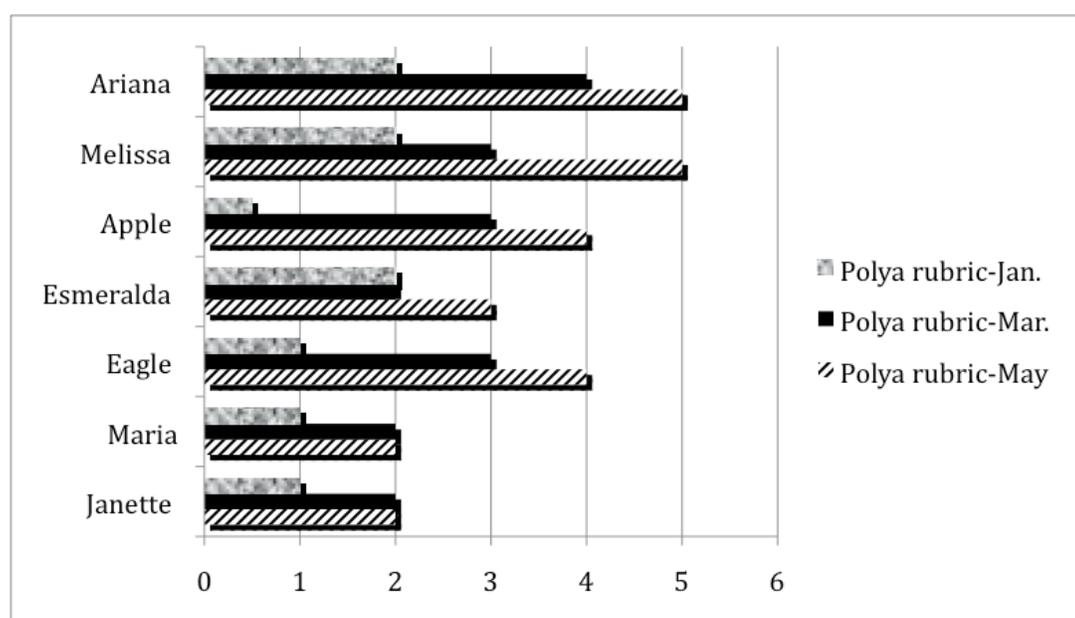


Figure 4.2. Polya's Problem-Solving Plan Average Scores

Round One Transcriptions

In the first round of retrospective reports, I transcribed my conversations with six sixth-grade participants as they described their thinking when solving two fraction word problems. As I indicated above, I labeled each of the transcriptions with numbers, which correspond to Polya's problem-solving steps. Appendix D includes the labeled transcriptions.

Three of the six participants used four of Polya's steps. One of six students was able to solve the problem only (step 4). Five of the six participants were able to demonstrate the first four steps in Polya's problem-solving plan.

Table 4.1

Numbered Coding of Polya's Steps

Polya's Steps	Step Number in Transcription
<i>Understand the Problem: Tell what the question is asking you to find</i>	1
<i>Understand the Problem: List what information you already have</i>	2
<i>Make a Plan: List all math steps you will need to take to solve</i>	3
<i>Solve the Problem: Do the math</i>	4
<i>Look back and check: Look back over your work for any errors</i>	5
<i>Look back and check: Use a strategy to check your work</i>	6

Five of the six participants successfully used step 1. Apple used this step when describing what information revealed the need for addition. When asked if any words inferred what he should do, he stated "how much, uh, altogether."

Four of the six participants identified the information they needed to find in order to solve their word problems. Melissa used an underlining technique to remind herself of the important information. When asked why she underlined three days three times, she responded, "because three days, is, they were saying like the three days like, um, like on Monday, Tuesday, and Saturday, they were the three days."

Five of the six students demonstrated their ability to list and describe the steps

they took to solve their word problems. When solving a fraction problem, Ariana stated:

I drew a circle, and it had three pieces in it. And I filled in one, one of the pieces, and then I put $1/8^{\text{th}}$. Then I drew another circle, and I drew 4 pieces in it, and I colored in 3 of them, and that equal $3/4^{\text{ths}}$. And then, so I know then the shaded pieces are 4 then.

All participants demonstrated that they were able to solve the problem by describing mathematical steps (*Step 4*). When finding the common denominator of two fractions, Maria explains

um, well, it can't be over fourteen so I um . . . put how many eights go into fourteen and it's one so I put one and then it's eight cuz it's always eight and then I put this fourteen and then fourteen minus eight is six.

When describing how she changed the denominators of fractions, Esmeralda said “and then I did 4 times 2, which is 8, and then I did 3 times 2, which is $6/8^{\text{ths}}$, and then I got $7/8^{\text{ths}}$.”

Apple was the only student to look back at his work and check for errors (*Step 5*). He paused and explained “--yeah. I think that I forgot to add these.” All other participants did not verbalize any errors they may have noticed in their work.

Three of the six participants demonstrated their ability to use a strategy that helped them check their work. Eagle chose to check his work by drawing pizza pictures saying “because there's like, um, 11, like one big circle that's a fraction. There's like one

whole pizza, so instead of that, there's 11 pizzas.” Because he could draw a picture, he knew his answer was correct. When asked how she might check her work, Ariana said “ummm / uh, k / like, list all the multiples, like make sure they all have 8s in them.”

Overall, the six participants showed a clear understanding of how to solve the problem, and were often able to describe their mathematical plan based on the information they already had and the information they knew they needed to find. In Table 4.2, each of the participants is listed with the steps they used in each round of retrospective reports.

Table 4.2

*Polya Steps Used by Participants in Retrospective Reports*²

Participants	Round 1-Steps Used	Round 2-Steps Used	Round 3-Steps Used
Maria	4	1 3 4 6	1 2 3 4
Eagle	1 3 4 6	1 2 3 4 5 6	1 2 3 4 5
Ariana	1 2 3 4 6	1 2 3 4 5	1 2 3 4 5
Esmeralda	1 2 3 4	1 2 3 4	1 2 3 4 5 6
Melissa	1 2 3 4	1 2 3 4 5 6	1 2 3 4 5
Apple	1 2 3 4 5 6	1 2 3 4 5 6	1 2 3 4 5 6

Round Two Transcriptions

In round two, the participants discussed word problems relating to area and perimeter. The comments made in round two were longer and more of Polya's steps were mentioned than in round one of the retrospective reports. As one can see from Table 4.2, all of the participants used at least four of Polya's steps when describing their

² [Janette moved before May data could be collected so she is not included in this data.](#)

thoughts on the two word problems given. All students were able to use the first four steps of Polya's plan at this stage.

All participants understood the information given and were able to tell what the question was asking them to find (*Step 1*). When Maria was asked how she knew to find several lengths and widths she responded, "well cuz the problem says to write down all the numbers you know." Ariana also showed her understanding of the question when she was asked how she knew what to do with the lengths and widths. She explained, "cuz it says the area and I know that area equals length times width."

Five of the six students identified the relevant information they already knew. When asked how she solved a word problem about area, Melissa described how she knew that "it said the floor area of 240 square feet" so she needed to use 240 square feet in her work.

All participants described the steps they would use to solve the problem (*Step 3*). Maria was able to describe her thinking when solving an area problem stating, "I was like thinking of what gives me 24 and then I added zeros on." As she described these steps she pointed to her notes showing the pairs of numbers she multiplied together to get 24.

Just as in round one, all participants were able to solve the math problems they were given during round two collections.

Four of the six students checked for errors while working on their word problems. While working, Ariana caught a mistake explaining, "I think I was supposed to do 240 divided by two to get 120 and 120 or something. So I am thinking that since it says that she wants to make it 240 and no bigger that I should divide instead." Melissa also caught

her mistake when determining possible lengths and widths, saying “I said 240 divided by something and I thought twelve and then I said twelve times ten equals 120 and then I thought that’s not even close.”

Four of the six participants also checked their work using a strategy. Eagle tried to visualize a rectangular area, explaining “I tried to divide it because it might be the other half of the width cuz 65 is the whole thing and if I divide it it might give me like the other side.”

All participants used the same number or more of Polya’s steps in round two than in round one. Three of the six students described their use of at least one more of Polya’s steps in round two than in round one. Melissa, for example, used steps 1-4 only in round one but added steps 5 and 6 in round two. Three of the six participants used the same number of steps in round two as they did in round one. Five of the six participants were able to not only repeat the use of the same steps from round one to round two but added additional steps in the second round. Maria, for example, used only step 4 in round one but added steps 1, 3, and 6 in round two.

Round Three Transcriptions

In round three, the participants’ described their thoughts about data analysis word problems. Their explanations became more succinct than the other two rounds and all participants used at least four of the six steps in Polya’s plan. Five of the six participants described their thinking using five of Polya’s problem-solving steps. Four of the six students increased or repeated the same number of steps they used from round two to

round three. All students were able to use the first four steps in Polya's problem-solving plan in this stage.

All the students improved their ability to describe what the problems were asking them to find (*Step 1*). Maria clearly described, "for the last one it said the mode and I think the mode means, like, the most (often)." When asked to find the average Eagle explained, "I read what is the relay team average time and I thought average means bigger number" which helped him to determine his first steps to the solution.

In order to make their answer more comprehensible to the listener, many of the participants were able to synthesize the first four steps of the problem-solving plan. Eagle, for example, combined Steps 1-4 when he stated, "the first question was what was the mean average high temperature for the five days and I put Tuesday, 93 degrees because on the high days, Tuesday was the hottest day in the week." In this sentence he stated the given information (*Step 2*), the information he needs to know (*Step 1*), the steps he took to solve the problem (*Step 3*) and the answer he got after solving (*Step 4*). Apple synthesized Steps 1-4 a when he responded, "in the last part, it asks for all the ten temperatures, so it asked for the mode, which is most often, and I found 79 because there are three." Not until round three did this type of synthesis take place.

Five of the six participants also looked back over their work and caught errors they had made while solving the problem (*Step 5*). Esmeralda caught her mistakes when solving for the mean. She thought she should subtract the lowest number from the highest but stated, "oh, wait. The lowest number is supposed to be 85 so I need to fix my answer." While only two of the six students looked over their work and used a strategy to

check their thinking (*Step 6*) in round three, all students showed an overall increase in the number of steps they used from round one to round three. Five of the six participants successfully used all of Polya's problem-solving steps at some point during the rounds of retrospective reports. Although there was an overall increase in the number of steps used, the participants did not show a consistent increase in the number of steps they described from round one to three.

Fluency

In order to determine whether the participants' improved their fluency when communicating orally about their thinking, I listened to the retrospective report recordings to count the number of pauses for each student in each of the three rounds of recordings. In Table 4.3, I noted the number of pauses for each student in each round of recordings.

Table 4.3

Fluency Test: Number of Pauses in Retrospective Reports

Participants	Round 1	Round 2	Round 3
Maria	29	17	12
Melissa	18	17	7
Eagle	34	21	8
Apple	22	19	7
Esmeralda	13	9	6
Ariana	30	19	9

In round one, all of the participants paused 13 or more times during their explanations. Four of the six participants uttered 22 or more pauses as they described their thinking immediately after solving the two word problems. Two of the students

paused 30 or more times throughout the course of their explanation. Eagle paused the most, making his speech choppy. The following is an example of his pausing pattern:³

First, you find / the LCM, first / You find the LCM first.
 Then, / find the LCM for 4 and 8. Like for 4, / it's 4, 8, 12.
 For 8, it's 8, 16. Then you find/the LCM, so that /the LCM
 is 8. Put the 8 under/the numbers /there. I explained it
 with numbers. Then, / I, / I, / I think/ "how / you get to 8?"
 / Times it.

In round two, all of the participants decreased the number of pauses when compared to round one. Half of the students decreased their number of pauses by 11 or more from round one to round two. Ariana reduced her pause count from 30 to 19 from round one to round two. Below is a comparison of her pause patterns in round one and round two:

Round One: I drew a circle /, and it had three pieces in it.
 And I filled in one, / one of the pieces, / and then I put
 1/8th. Then I drew another circle, / and I drew 4 pieces in
 it, / and I colored in 3 / of them, and that equal 3/4ths. And
 then, / so I know then the shaded pieces are 4 then. And
 the remain/ing ones, / there's / 6, so it's four sixths.

³ / = pause of three or more seconds

Round Two: It said in the question what is the perimeter / and what is the width of the storm shelter / and um, / I knew right away that the width meant the area, well not the area but the space around it / and so the perimeter is like when you add up all the sides and so it said that the storm shelter was 65 square meters and the length was six and a half meters / and so I knew right away that I had to find the perimeter which you have to add all the sides and I did 65 plus 65 and that would equal 143, the perimeter. For the area I did length times width / so six and a half is the length and 65 square meters is the width.

In these examples, it is evident that Ariana decreased the frequency of her pauses from round one to round two.

All of the participants also decreased their number of pauses from round two to round three. Four of the six participants decreased their pausing by 10 or more from round two to three. Melissa reduced her number of pauses from 19 to 9. Below are examples of how her pausing pattern changed from round two to round three:

Round Two: So first I said / 240 divided by something and I thought twelve / and then I said twelve times ten equals 120 / and then I thought that's not even close so I put 12 times 20 equals 240 and that how I get 20 feet on the length / and on the width / I get 12 feet. I checked by multiplying

20 times 12 / and I got 240 feet squared so it was good.
Then I knew I had to find the perimeter which is 64 feet
cuz / I add all of them.

Round Three: For part a, it said / what is the relay team's
median practice time. I crossed out the numbers / and then
the median one is 3.39. First / I put them in order. Then in
part b, I add all of the times because it says average and
then I got 17.4 cuz I add all of them. In part C, it said what
is the outlier. I said 3.89 is the outlier because it's like / out
and far away from the other numbers and 3.14 isn't the
outlier because it isn't as far away from the other numbers.

Overall, these data show that the participants' fluency increased due to the consistent reduction in the number of pauses from round one to round three.

Vocabulary and Content Knowledge

As I began to analyze the transcriptions from the participants' retrospective reports, I found that many of the issues they encountered when solving word problems had to do with their lack of background knowledge in content-specific vocabulary. Often student vocabulary issues were clear even when reading the word problems aloud. Students would stumble on math vocabulary words they did not know or could not remember.

In round one of the retrospective reports, students were asked to solve fraction word problems. All participants were fairly comfortable with general fraction vocabulary such as *common denominator* and *least common multiple* but struggled with the pronunciation and comprehension of word names for fractional parts such as one-half or two-eighths. Because they stumbled while describing these fractional parts, the meaning was sometimes changed for the listener. Maria, for example, struggled most with these fractional words as she tried to explain her steps saying, “six and a half and 4 and I don’t know how to say it.” She also said phrases such as “two and one two” instead of two and one-half, which could be confused by a listener who was unfamiliar with this error.

In round two, the participants struggled more with content vocabulary that they had not yet mastered. When asked to solve word problems on the area and perimeter of rectangles, several students failed to recall the meaning of words like width and length and sometimes confused the definitions of area and perimeter. This confusion not only affected their mathematical accuracy but also their ability to intelligently communicate their ideas about the problem.

In round three, the students who lacked content knowledge in data analysis problems struggled with remembering the difference between mean, mode, range and median. Ariana, for example, explained “In part b it says, what is the relay team’s average time and I put Wednesday, 3.89 because it was the most and the average means like the most, the biggest.” Esmeralda also had a hard time distinguishing between these vocabulary terms and stated; “I organized the big number to the small number but I wasn’t sure what average meant. So I tried to subtract the biggest number minus the

smallest number cuz I remember that from math.” Both students were unsure of the meaning of these math-specific vocabulary terms, which stunted their thinking and made their responses more difficult to understand.

Conclusion

In this chapter, I have presented the results of my data collection. In summary, the results showed that 100% of the participants increased the number of Polya’s problem-solving steps they used when explaining their mathematical thinking in writing. While the participants were using more of Polya’s steps in the second and third rounds of testing, their average constructed response scores did not consistently increase. The oral retrospective recordings showed that overall, 83% of the participants increased the number of Polya’s steps used while explaining their thinking from rounds one to three. However, the increase in the number of steps used from round to round was inconsistent. The results revealed that 100% of the participants consistently decreased the number of pauses in their mathematical explanations from round one to round three of the retrospective reports. In Chapter Five I will discuss my major findings, their implications and my suggestions for further research.

CHAPTER FIVE: CONCLUSIONS

In this study, I attempted to determine whether Polya's problem-solving plan was an effective strategy for improving middle school ESL students' oral and written communication of their mathematical thoughts when solving word problems. I also sought to find out if the use of Polya's plan improved ESL students' fluency when describing the steps they took to solve a word problem. In this chapter, I will discuss my major findings, limitations of this study, implications for teachers and administrators, and suggestions for further research.

Major Findings

Increased Use of Polya's Steps One through Four

One key guiding question I tried to answer was whether ESL students used more of the steps in Polya's problem solving plan over time. While the participants were inconsistent in their use of Polya's problem solving steps in oral tests, they did increase their use of steps one through four from 66.6% in round one to 100% in round three. Some participants were already using steps one through four at the start of this study but by the end of the study, all students successfully implemented steps one through four. This demonstrates that after the repeated use and modeling of Polya's problem-solving strategy, ESL students are more likely to understand the problem and find both missing and available information, make a plan of action, and successfully complete their math

plan when solving word problems. The participants were less successful with their use of steps five and six. This inconsistency could be the result of the need for more examples of math strategies that can be used to check for errors in a problem or the participants' lack of understanding of the mathematical topic they were studying at the time of the data collection.

While Chamot, et al. (1992) did complete research on the use of Polya's problem solving plan with ESL students, they did not comment on specific steps of the plan that were most widely used by English Language Learners (ELLs). It is difficult to compare my findings to other research done on Polya's plan and the use of step one through four because I was unable to find any other similar studies.

Math Content Knowledge Key to Accuracy

Another question I wanted to answer was how the use of Polya's problem-solving plan affected middle school ELLs' ability to solve mathematical word problems. In the MCA-II rubric results, participants did not show a consistent improvement in the rubric score from round one to round three but six of the seven participants increased their rubric score from January to May. This would imply that the use of Polya's problem solving plan does not improve all ESL students' ability to solve word problems. Since most participants did improve their rubric score, however, it seems that using Polya's problem-solving plan may be helpful for most students. When scoring the MCA-II word problem responses, it became clear that many of the errors were made because of a lack of understanding of the math topic being tested. The accuracy of the mathematical work had less to do with the use of Polya's steps and more to do with the background

knowledge the students had on the particular math topic. This finding works in combination with Schleppegrell's (2007) ideas about the unique content language and structure of mathematical text. Carr, et al. (2009) and Cuevas (1984) also agree that math-specific vocabulary terms add another dimension of linguistic difficulty for ELLs. When viewing the change in MCA-II rubric scores from round one to round two, it is apparent that five of the seven students earned the same or lower average score in round two than round one. After listening to the participants' retrospective reports, it was clear that these students were struggling more with the word problems on area and perimeter than they were with the fraction problems. The struggle occurred because the students did not have a good understanding of the formulas for area and perimeter. They were confusing the meanings of these words, which in turn affected their accuracy when solving the problem. Basurto (1999) also found that vocabulary and language comprehension play a huge role in mathematical understanding for ELLs. My findings work in coordination with Basurto's in that both sources point to a need for a focus on language skills along with the teaching of a problem-solving plan.

The participants' lack of background knowledge also affected their use of Polya's steps in their retrospective reports. When students' struggled to understand the meaning of vocabulary words in the problem, they were also unsure of how to look for errors (*Step 5*) and did not know which math strategy they should use to check their work (*Step 6*). As I explained in chapter four, participants had difficulties when recalling the definitions of terms such as mean, median, mode and range. If students had not yet mastered the definitions of these vocabulary terms, they became confused and could not properly solve

or check their work. Their difficulties when defining unknown vocabulary words can be seen in all rounds of transcriptions. In round two for example, Esmeralda was confused about the definition of width [sayingsaying](#). “the problem says we need to find the width. I think that’s six and a half cuz it says the length is six and a half and that is like the same as the width.” Eagle also struggled with vocabulary in round three when he did not remember the meaning of average stating, “I read what is the relay team average time and I thought average means bigger number and so I saw 3.89.”

Fluency Improved with the Use of Polya’s Steps

A major question I had at the onset of this study was how Polya’s problem-solving plan might affect an ESL students’ fluency when describing their thoughts both orally and in writing. I was pleased to notice a large improvement in the participants’ fluency from round one to round three. The most drastic improvement in this study came when I determined the number of pauses in each round of the retrospective reports. All participants decreased the number of pauses made from round one to round two and from round two to round three. The transcriptions were much shorter as a result of the participants’ more organized approach to describing their thinking. Polya’s problem-solving steps served as an organizational strategy for the students and helped them to pull out key information needed to solve the problem before they began their explanations. Because their responses were less choppy and more succinct, their messages were better understood. All of these results point to a clear improvement in fluency for all participants as they used more of Polya’s problem-solving steps.

Other research is in agreement with my findings, stating that instruction in a problem-solving strategy will improve metacognition and a student's ability to communicate his or her mathematical thoughts (VanSciver, 2008; Bender, 2005; Van de Walle, 2004). Other researchers agree that explicit teaching of a problem-solving plan leads to greater success when students are asked to work on problems independently (Bender, 2005; Schoenfeld, 1992 as cited in Van de Walle, 2004; VanSciver, 2008). These studies work in coordination with my findings to indicate that instruction in Polya's problem-solving plan or a similar problem-solving plan will help intermediate and advanced ESL students to more clearly communicate their thinking when solving math word problems.

Polya's Plan Used More in Oral Communication

At the beginning of this study, I wondered whether middle school students would use Polya's problem-solving plan when they were asked to solve math word problems. After analyzing both the MCA-II response data and the retrospective report data, it seems clear that students chose to use Polya's plan on all word problems. When comparing the use of Polya's steps in the retrospective reports to the MCA-II written response data, it becomes apparent that the participants used more of Polya's steps when describing their thinking out loud in the retrospective reports. In the retrospective report data, 66.6% of the students used all six of Polya's steps in one of the rounds of data collection. In their written responses, however, none of the participants ever used all six of the steps and only two students ever used five. Several researchers found results similar to mine when testing ELLs as they solved math word problems in writing (Martiniello, 2008; Lager,

2006). All of the research on high-stakes written response exams and ELLs points to their difficulty when expressing their mathematical thoughts in writing.

Huang and Normandia (2007) agree that students' understanding and use of language structures in mathematics are a direct indication of how well the students will be able to communicate their mathematical ideas in writing. Their results showed how written and oral communication are linked when students are conveying mathematical thoughts. My findings did not create an explicit link between fluency in written and oral communication.

Huang, Normandia, and Greer (2005) also found that even native English speakers conveyed more fluent oral answers when they were taught how to think aloud. These results are consistent with my findings in that they suggest that students' oral communication will become more fluent as they become more comfortable with the use of some type of problem-solving strategy. Because their research focused on the fluency of written responses separately from oral responses, it is impossible to determine which mode of communication allowed students to produce more fluent mathematical responses.

Implications

While this study cannot prove that the use of Polya's problem-solving plan is the sole reason for improved oral fluency, it does show that it is a major factor in the improvement of ESL student communication when solving word problems. The study also demonstrates that Polya's plan helps ESL students organize their thinking before trying to convey their message. Finally, the study notes that the teaching of Polya's

problem-solving strategy will improve ESL students' abilities to understand what information is important in a math word problem and what information students still need to find.

Students Need Organizational Strategy

This study demonstrates the importance of providing ESL students with organizational steps or techniques, which can help them to prioritize their thoughts and communicate them effectively when solving word problems. At the beginning of this study, the participants conveyed their thoughts in an unstructured and choppy way, with many pauses. This often leads to a comprehension issue for listeners. Strategies like Polya's plan are effective because they give ESL students an organizational tool that they can use before they even start communicating their message out loud. They can use the strategy to organize their thoughts on paper or in their mind and then convey a much more structured and fluent message.

Importance of Concept Knowledge with Polya's Steps

In this study students did not show improved mathematical accuracy with the use of Polya's problem-solving plan alone. They struggled because of a lack of background knowledge in the particular math topic they were studying. It is crucial that content teachers include both language-rich vocabulary and content instruction along with the modeling of Polya's plan. ESL students need more work with key vocabulary and concept knowledge in order to accurately solve a math problem. They need to understand the background in order to manipulate the known information.

Focus on Oral Communication

This study suggested that ESL students were much more likely to use Polya's problem-solving steps when they were asked to explain their thinking orally. It seems clear then that ESL students would benefit from more think-aloud modeling by their content teachers as a way of training students to use Polya's steps. After exposing ELLs to this kind of instruction, they could be assigned to try using the steps in an exercise with their peers where they are required to think aloud. If oral communication became a regular way of training students to use Polya's steps, content teachers would hear more fluent and structured answers to word problems. Teachers could then connect the oral instruction and practice to a written response with more modeling. This would be a great way to connect a more natural way of communicating with the formal written responses they will eventually need to produce.

Limitations

One limitation of this study is that it did not begin at the start of the school year. Because it did not start in September, students may have been exposed to Polya's problem-solving steps in their other content classes before they began the study. Students may have also heard reference to a more general problem-solving plan in math class as we worked on word problems. The students were also accustomed to my teaching style and method of explanation prior to the study. This exposure may have aided them in the first round of retrospective reports because it might have given them an understanding of what I was looking for in their explanations.

A second limitation of the study might be that I used a small group of participants. Because I used only seven participants, my data did not allow for inferential statistics to be done. My data did, however, allow me to do rich analysis on the detailed interview transcriptions.

A final limitation is that I focused my study solely on Polya's problem-solving plan as a factor in improving oral and written communication when describing word problems. Because my participants were sixth graders, it is possible that other factors such as motivation to learn, shyness when being recorded, and interest in math played a part in the quality of their responses.

Further Research

While working through this study, several questions arose that would benefit from further research. I discovered that even with the use of Polya's problem-solving steps, students struggle with vocabulary terms when they do not possess sufficient background knowledge in a particular mathematical topic. Carr, et al (2009), Johnson (2010), and Cuevas (1984) discovered that math vocabulary is a difficult area for ELLs because the content-specific meanings of words could be quite different from the words' everyday use. I wonder which vocabulary teaching strategies might work best in coordination with Polya's problem-solving plan. I also wonder if the teaching of reading strategies might help support vocabulary and concept understanding while students are solving math word problems. Basurto (1999) found that ELLs benefit from reading strategies like prediction and questioning when they are working through math word problems. Chamot, et al.

(1992) did some research in this area but more research could be done to determine which reading and vocabulary strategies work best with Polya's problem-solving plan.

Another question I came across is whether or not research has been done in other states on oral responses to word problems and the possibility of grading oral responses as an ESL modification to word problems on state tests. Martiniello (2008), VanSciver (2008), and Lager (2006) tested elementary and high school ELLs with standardized written tests and found that there is no way to test their math abilities without also testing their understanding of language. The Center on Instruction has done research on the need for ELL accommodations on state tests so that ELLs are able to understand what is being asked while at the same time testing their concept knowledge (Francis, Rivera, Lesaux, Kieffer, & Rivera, 2006). Many of the accommodations currently used by states have not been properly researched and have not proven to be helpful to ELLs. Some states offer oral accommodations for ELLs, such as the directions being read in a student's native language or dictation of a student's answers by a scribe, but little work has been done to determine the effectiveness of these accommodations. Read-aloud accommodations were studied with ELLs but showed different results in different states (Wolf, Kim, Kao, & Rivera, 2009). Since my study suggests that middle school ESL students' oral communication is more fluent than their written communication, it is clear that more work should be done to consider whether an oral recording accommodation might be a better option for ESL students when taking the state mathematics exam.

Because no state currently offers an oral exam accommodation for ELLs, it is important that teachers work to find effective accommodations that might correspond

well with student performance on a written test. Francis, et al. (2006) studied the effects of seven different accommodations currently used on state tests. They found that the use of English-language dictionaries along with the allowance of extra time on the test showed the biggest positive change in ELL test scores. Even this accommodation, however, did not show significant improvement especially among ELLs who were illiterate or had little understand of the structure of dictionaries. Abedi, Hofstetter, and Lord (2004) studied several groups of ELLs and found that even accommodations such as the use of simplified language or bilingual dictionaries do not have a significant impact on the performance of ELLs on state tests. This study led me to wonder what other accommodations teachers might suggest to promote ELL success on the current state tests. Based on my findings, it might be helpful for teachers to post Polya's problem-solving plan in their room while students take their test. This strategy would allow students to organize their thinking on their scratch paper using Polya's steps. ELLs would also benefit from specific content and vocabulary instruction targeted at building background knowledge in mathematics. Teachers should post a list of math strategies they have taught throughout the year so students can refer to them as they take their test.

In order to pass on the information I have learned about math word problems and ELLs, I plan to meet with my colleagues who teach math prior to the start of the next school year. As we plan our lessons for the year, I will share my findings and encourage our continued use of Polya's problem-solving plan. I will discuss the results of my study and explain the need for more focused work on building conceptual knowledge. I hope that my colleagues and I are able to make a list of math-specific strategies we can all

agree to use when teaching math content in our classrooms. I will also share vocabulary strategies I have collected over the years and describe the errors and language issues I noticed as I listened to the retrospective reports. I will send an electronic link to my capstone once it is published on Hamline's webpage to both my colleagues in the math department and my district administrators. My intention is to spread the word about the improvement ELLs make in their fluency after using Polya's problem-solving plan so that this plan will be used district wide. I will also suggest the use of this plan to any ELL teachers who are looking for effective ways to improve ELL communication in math. Because I enjoy sharing and collaborating with other teachers, I hope to present my findings at the Minnesota Teachers on English as a Second Language (TESOL) conference in 2011.

This study was a significant undertaking, especially because I chose to complete three data collection periods. Even though this extensive collection period took a large amount of time, I feel satisfied because I was able to report some important results due to the rich retrospective report recordings. I feel this capstone also gave me an opportunity to finally research and investigate on a topic I have always wanted to know more about. I have been the Mathematics Test for English Language Learners (MTELL) school coordinator for the last four years. This experience led to my interest in state testing modifications and other related research. After reviewing several different versions of MCA-II math tests and MTELL tests and talking to students about their struggles, I became more aware of the need for more modifications and data analysis. Before this project, it seemed that I never had a chance to truly dive into data contrived from my own

students. Through deep analysis, I was able to understand where some of my students' struggles with word problems began and how certain instructional practices were truly affecting their performance and understanding. I honestly feel that this process has made me a more thoughtful ESL and math teacher and will forever affect the way I analyze student data as I plan my lessons. I will be able to take the information I have learned about effective testing modifications and apply it to the lessons I teach on test preparation.

APPENDIX A

MCA-II Constructed Response Questions

Round 1- January, Question #1

Juice boxes are packaged two different ways.

- A package of 24 boxes costs \$12.98.
- A package of 4 boxes costs \$2.59.

You need 56 juice boxes.

Part A Find 2 different possible ways you could purchase the 56 juice boxes and the total cost for each way. Show and explain your work.

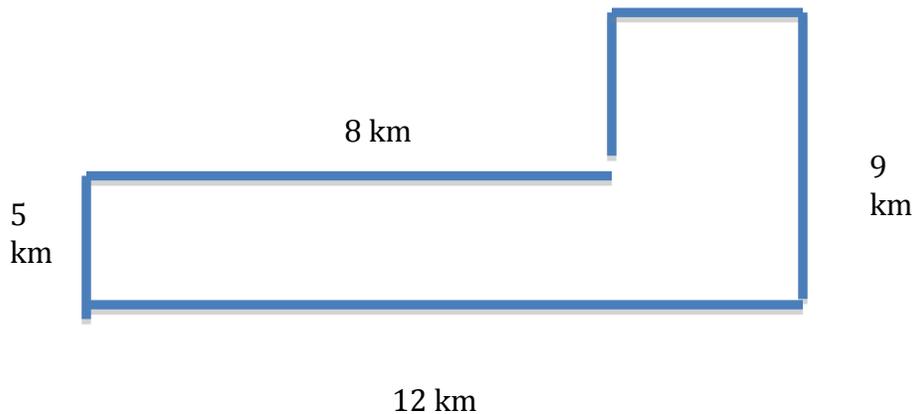
Part B Using your answer in part A, what is the price per juice box if you purchase the lower priced combination? Show or explain your work.

Round 1-January, Question #2

Expenses	Fractional Part of Paycheck
Rent	$\frac{1}{3}$
Car Payment	$\frac{1}{5}$
Food	$\frac{1}{5}$
Household Bills	$\frac{1}{10}$

The table above shows 4 of Victor's monthly bills.

- What fractional part of his paycheck does he spend on Rent and his car payment?
- What fractional part of his paycheck does he spend on all 4 expenses?
- How much more of his paycheck does Victor spend on Rent than on Food?

Round 2- March, Question #1

Part A. Heidi wants to find the distance around her school. Find the total distance around the school.

Part B. What mathematical term do we use to describe the distance around the school?

Part C. Heidi also wants to find the area of the school. What steps should she use to find the area?

Write your answers in the space below and on the back of this paper.

Round 2- March, Question #2

Karen has rectangular garden that is 16 feet long and 12 feet wide.

Part A Karen builds a fence around the perimeter of garden. How many feet of fencing does she need? Show or explain all of your work.

Part B Karen plants tomatoes in 40 square feet of the garden. Remember the garden is 16 feet by 12 feet. What are a possible length and width of the area where Karen plants her tomatoes? Show or explain all your work.

Round 3- May, Question #1

The students in Mrs. Jones's class took a geometry test. Their scores were the following: 89, 88, 89, 91, 94, 95, 76, 74, 67, 72, 70.

Part A: What is the mean score of the tests? Show your work.

Part B: Is there an outlier? Explain your thinking.

Part C: What is the range of the scores? Show your work.

Round 3- May, Question #2

Adam has a bowl that contains gumdrops of all the same size. The bowl contains these gumdrops:

- 10 green
- 10 red
- 10 yellow
- 5 purple

Adam takes one gumdrop from the bowl at random. Explain your answers for each of the questions below.

Part A: What is the probability that it will be purple?

Part B: What is the probability that it will be red?

Part C: What is the probability that it will be red or green?

APPENDIX B

MCA-II Constructed Response Rubrics

Round 1- January, Rubric #1

**MCA-II Item Sampler
Rubric, Sample Responses and Answer Annotations
Grade 6 Mathematics**

Score	DESCRIPTION
4	Response contains correct solutions for: <ul style="list-style-type: none"> • 2 different possible ways you could purchase the 56 juice boxes • what would be their total costs • the price per juice box The responses adequately show or explain all work.
3	Correctly determines the answer to at least three questions with adequate supporting work OR Correctly determines the answer to all questions with incomplete supporting work.
2	Correctly determines the answer to at least two questions with adequate supporting work. OR Correctly answers at least three questions with incomplete or missing work.
1	Some work relevant to the problem.
0	Response is incorrect or irrelevant.

Sample Response:

A. 2 24-packs at \$12.98 each and 2 4 packs at \$2.59 each for \$31.14

OR

14 4 packs at \$2.59 each for \$36.26

OR

1 24 pack at \$12.98 and 8 4 packs at \$12.98 and 8 4 packs at \$2.59 for \$33.70

B. the price of the less expensive of the two combinations divided by 56; for example,
 $\$31.14 / 56 = \text{about } \0.56 each

Round 1-January, Rubric #2

**MCA-II Item Sampler
Rubric, Sample Responses and Answer Annotations
Grade 6 Mathematics**

SCORE	DESCRIPTION
4	Response contains correct solutions for: <ul style="list-style-type: none"> ● the total amount spent on Rent and car payment ● total amount spent on rent, car payment, food and household bills ● the difference between the amount spent on rent and food The responses adequately show or explain all work.
3	Correctly determines the answer to at least 2 parts with adequate supporting work. OR Correctly determines the answer to all questions with incomplete supporting work.
2	Correctly determines the answer to at least 1 part with adequate supporting work. OR Correctly answers at least 2 parts with incomplete or missing work.
1	Some work relevant to the problem.
0	Response is incorrect or irrelevant.

Sample Response:

$$\begin{array}{r}
 \text{A.} \quad \frac{1}{3} = \frac{5}{15} \\
 + \quad \frac{1}{5} = + \frac{3}{15} \\
 \hline
 \qquad \qquad \frac{8}{15}
 \end{array}$$

$$\begin{array}{r}
 \text{B.} \quad \frac{1}{3} = \frac{10}{30} \\
 \quad \quad \frac{1}{5} = \frac{6}{30} \\
 \quad \quad \frac{1}{5} = \frac{6}{30} \\
 + \quad \frac{1}{10} \quad + \frac{3}{30} \\
 \hline
 \quad \quad \frac{25}{30} \qquad \text{OR} \quad \frac{5}{6}
 \end{array}$$

Round 2- March, Rubric #1

**MCA-II Item Sampler
Rubric, Sample Responses and Answer Annotations
Grade 6 Mathematics**

SCORE	DESCRIPTION
4	Response contains correct solutions for: <ul style="list-style-type: none"> ● the definition of perimeter ● the area of the school (in squared km) ● the perimeter of the school (in km) The responses adequately show or explain all work.
3	Correctly determines the answer to at least 2 parts with adequate supporting work. OR Correctly determines the answer to all questions with incomplete supporting work.
2	Correctly determines the answer to at least 1 part with adequate supporting work. OR Correctly answers at least 2 parts with incomplete or missing work.
1	Some work relevant to the problem.
0	Response is incorrect or irrelevant.

Part A. $12\text{km} + 5\text{ km} + 8\text{ km} + 4\text{ km} + 9\text{km} = 38\text{ km}$

Part B. perimeter

Part C. $(4\text{ km} \times 5\text{ km}) + (12\text{ km} \times 5\text{ km}) = (20) + (60) = 80\text{ km}^2$

Round 2- March, Rubric #2

**MCA-II Item Sampler
Rubric, Sample Responses and Answer Annotations
Grade 6 Mathematics**

- Correct number of feet in A
- Correct work (procedure) shown or thinking explained in A

- Correct possible length and width in B
- Correct work (procedure) shown or thinking explained in B

Score	DESCRIPTION
4	4 of 4 parts correct
3	3 of 4 parts correct
2	2 of 4 parts correct
1	1 of 4 parts correct OR Some correct and relevant work or explanation
0	Response is totally incorrect or irrelevant

Sample Response:

- A.** 56 (feet)
AND
 $16 + 12 + 16 + 12 = 56$ equivalent (e.g., $2[16 + 12]$)
- B.** Any length and width such that length times width = 40

For examples:

Length in feet	Width in feet
10	4
8	5

Round 3-May, Rubric #1

**MCA-II Item Sampler
Rubric, Sample Responses and Answer Annotations
Grade 6 Mathematics**

Sample Responses:

Score	DESCRIPTION
4	Response contains correct solutions for: <ul style="list-style-type: none"> • probability of purple gumdrops • probability of red gumdrops • probability of green and red The responses adequately show or explain all work.
3	Correctly determines the answer to at least three questions with adequate supporting work OR Correctly determines the answer to all questions with incomplete supporting work.
2	Correctly determines the answer to at least two questions with adequate supporting work. OR Correctly answers at least three questions with incomplete or missing work.
1	Some work relevant to the problem.
0	Response is incorrect or irrelevant.

Part A: $5/35 = 1/7$

Part B: $10/35 = 2/7$

Part C: $10/35 + 10/35 = 20/35 = 4/7$

Round 3- May, Rubric #2

**MCA-II Item Sampler
Rubric, Sample Responses and Answer Annotations
Grade 6 Mathematics**

Sample Responses:

Score	DESCRIPTION
4	Response contains correct solutions for: <ul style="list-style-type: none"> • mean score of tests • explanation for outlier • range of scores The responses adequately show or explain all work.
3	Correctly determines the answer to at least three questions with adequate supporting work OR Correctly determines the answer to all questions with incomplete supporting work.
2	Correctly determines the answer to at least two questions with adequate supporting work. OR Correctly answers at least three questions with incomplete or missing work.
1	Some work relevant to the problem.
0	Response is incorrect or irrelevant.

Part A: total divided by 11= 82.2

Part B: No, there are no outliers.

Part C: $95-67 = 28$

APPENDIX C

Retrospective Report MCA-II Questions

Round 1-January, Question #1

Suppose you eat $\frac{3}{4}$ of a pizza and then eat $\frac{1}{8}$ of another pizza of the same size. How much of a whole pizza do you eat altogether?

Round 1- January, Question #2

Evelyn rode her bike $6\frac{1}{2}$ miles on Monday. On Wednesday, she rode $2\frac{1}{4}$ miles. On Saturday, she rode $3\frac{1}{8}$ miles. How many miles did Evelyn ride in the three days?

Round 2-March, Question #1

The floor area of a rectangular storm shelter is 65 square meters and its length is $6\frac{1}{2}$ meters. What is the width of the storm shelter? What is its perimeter?

Round 2-March, Question #2

Alyssa is designing a garage with a rectangular floor area of 240 square feet. List all of the possible lengths and widths in feet that Alyssa could make. Use whole numbers in your answers.

Round 3-May, Question #1

Relay Team's Practice Times

Day	Time (min.)
Monday	3.24
Tuesday	3.48
Wednesday	3.89
Thursday	3.39
Friday	3.14

Part A: What is the relay team's median practice time?

Part B: What is the relay team's average time?

Part C: Is there an outlier in this data? Explain your answer.

Round 3-May, Question #2

Day	High	Low
Monday	91	79
Tuesday	93	79
Wednesday	79	63
Thursday	85	65
Friday	88	68

The table shows the high and low temperatures recorded for 5 days.

Part A: What was the mean (average) high temperature for the 5 days? Show or explain your work.

Part B: What was the median low temperature for the 5 days? Show or explain your work.

Part C: What was the mode for the 10 temperatures in the table? Show or explain your answer.

APPENDIX D

Transcriptions from Retrospective Reports- Polya's Steps Included

Key: / = pause of 3 seconds or more

Round 1-January

Maria's Recording:

Teacher: Okay, so I am going to read or I'm sorry, I am going to give you two problems and they're both about fractions and you're gonna look at the problem, read it, either to yourself or out loud, whichever you want, and then you're gonna write down any work that you would do, any math you would do to solve the problem and then once you are done writing I'm gonna say "okay, can you tell me what you were thinking. Okay? So, you can go ahead and start / with the reading of the problem out loud or to yourself.

Student: Wait, what do I have to do?

Teacher: Okay, so / you need to read the problem and then write down any math steps that you would use to solve to get the answer, okay?/ and then you're gonna tell me out loud. And you don't have to write in complete sentences. You can just write whatever math you do.

Student: yeah, I / yeah. [*points to steps written on paper- no math done*]

Teacher: Okay, can you show me with the actual numbers like actually get the answer?

Student: Yeah, it's not gonna work out.

Teacher: Just try it. Do the best you can.

Student: um / huh! [*confused about what to do next*]

Teacher: Well, do you wanna tell me what you're doin' so far? Go ahead and tell me what you're stuck on.

Student: Umm / I found the least common multiple is eight and then times eight times the one is eight and times / times twooh wait, times three / three times two is six and then it's eighths. [*Step 4*]

Teacher: How did you know that the LCM was eight?

Student: because it was twelve / it was like cuz that was like, they have it has I mean, they have the least common multiple. [*Step 4*]

Teacher: Okay, of which two numbers? What numbers in the problem are you lookin' at?

Student: The bottom ones cuz I knew that four would go into eight and eight goes into eight. *[Step 4]*

Teacher: Okay, gotcha okay cool, and then you got six because you did three times two?

Student: Yep.

Teacher: and how did you know to times by two?

Student: because four times two equals eight.

Teacher: four times two equals eight okay, cool. So then, do you want to keep solving so you can get your answer?

Student: Do I just take one away from it or minus it?

Teacher: Ah, I don't know, what do you think? Can I ask you a question? Can you tell me again how you got eight-eighths for that second fraction?

Student: Wait, what?

Teacher: Is this the new version of this fraction?

Student: Yeah.

Teacher: Okay. Okay, go ahead and continue. So the eight-eighths is the new version of one-eighth. Okay.

Student: I'm done.

Teacher: Okay, so how did you get your answer? What did you do?

Student: Um, well, it can't be over fourteen so I um / put how many eights go into fourteen and it's one so I put one and then it's eight cuz it's always eight and then I put this fourteen and then fourteen minus eight is six. *[Step 4]*

Teacher: fourteen minus eight is six, okay. So you took the top number and you minused eight to get the six which is the new top number.

Student: Yes.

Teacher: Okay. And then where did this one come from?

Student: Um, cuz it equals one whole.

Teacher: Okay. Great. Thanks. So I'm just gonna give you one more. It's the same kind of thing. It has fractions in it. Okay, and you're gonna do the same thing, you're gonna read the problem. You're gonna solve it, write down your steps and then you're gonna tell me out loud / owwww and I have one quick question about that last problem we just did, okay? So how did you know to add these fractions?

Student: Because we've been doin' it in class and /

Teacher: Okay. Is there anything in the problem though that told you to add?

Student: Yeah, it says how much.

Teacher: How much, okay. But did you just do adding because that's what we've been working on in class?

Student: Yeah.

Teacher: Okay. Have we been working on subtraction too?

Student: Yeah.

Teacher: How come you did subtract?

Student: I don't know.

Teacher: Alright, so try to second one here. Okay so go ahead and read it.

Student: I'm done.

Teacher: Are you done? Okay. Can you walk me through it, tell me what you did?

Student: Well, I first took these two numbers.

Teacher: Which are what?

Student: six and a half and 4 and I don't know how to say it. [*Step 4*]

Teacher: a fourth

Student: Yeah.

Teacher: uhhuh.

Student: and I put those two one and a half and one fourth and found the least common multiple and then *[Step 4]*

Teacher: Which was what?

Student: four

Teacher: okay and then tell me how did you get those / those new fractions?

Student: I added four times one is four and four times two is two and then I minused it which is two fourths and then I minused the six and the two which is four and then I added and then I put one eighth and *[Step 4]*

Teacher: Sorry, what's six and two?

Student: Six and two, six and a half.

Teacher: Where's the six and two, which numbers are you talking about?

Student: these *[pointing to whole numbers]*

Teacher: oh, the big numbers? Okay. Got it okay and you got the four and then what did you do?

Student: and then I put one eighth and two fourths and then I got and then the least common multiple was eight and then I got and then I added then times them by one and two and then it equaled five eighths when I minused four eighths and then one eight over eight and then I got five eighths and then *[Step 4]*

Teacher: SO eight over eight minus four over eight is five eighths?

Student: Yeah.

Teacher: Okay.

Student: and then I put four and three which equals one and it's one whole and five eighths. *[Step 4]*

Teacher: So you took this four from the first two numbers and minused the three from the last fraction? Is that right?

Student: Yeah.

Teacher: Okay and then that's how you got the one?

Student: Yeah.

Teacher: So the four came from the first two mixed numbers and the three came from the last mixed number. Am I getting that right?

Student: Yeah.

Teacher: Okay. So what's your final answer then?

Student: one and five eighths.

Teacher: Okay, alright, thank you very much Maria.

[Only Step 4 (do the math) used]

Eagle's Recording:

TEACHER: Alright. So, you are going to read the problem, think about it, you can read it to yourself or out loud. Then you're gonna write all the steps you were thinking as you were solving the problem and getting your answer. Then after you're all done writing down what you think, then you tell me each step that you did when you were thinking in your head. Okay? Go ahead. Start!

EAGLE: So I list all the steps?

TEACHER: Yup! So read it first, then show me—go ahead, write down your steps.

EAGLE: Can I like, write like this? Like the 4 and the 8?

TEACHER: You can, yup, you can write it however you want, but make sure that you're solving the problem, getting your answer, doing the math.

EAGLE: Please how is something spelled? Like, help me with the spellings?'

TEACHER: Don't worry about the spelling. In fact, you don't even have to write sentences if you don't want to. You can write just the math steps that you did.

EAGLE: Oh, like in math class?

TEACHER: Umhmm! Yup, you can write it however, however you want to solve it is fine.

[listing steps- step 3]

EAGLE: Thank you ma'am.

TEACHER: It's ok. You can do that starting now, if you want.

EAGLE: Erase this?

TEACHER: No, that's fine! It's beautiful!

EAGLE: I don't know how to put them in order, but I know how to do—

TEACHER: Then just show me, show me what you know first, it's fine. Did it?

EAGLE: Do you have to, like, simplify it?

TEACHER: I don't know, do you? I just mean are you done doing it? Are you ready to explain to me what you did. *[student nods]* Okay! Go ahead!

EAGLE: First, you find the LCM, first / You find the LCM first. Then, find the LCM for 4 and 8. Like for 4, it's 4, 8, 12. For 8, it's 8, 16. Then you find the LCM, so that the LCM is 8. Put the 8 under the numbers there. I explained it with numbers. *[Steps 3 and 4]*

TEACHER: Okay. So, yeah, so what did you do here to go from the $\frac{3}{4}$ s to the $\frac{6}{8}$ ths?

EAGLE: Um, I put the 8 under these two—

TEACHER: Yup, then?

EAGLE: Then, I, I, I think "how you get to 8?" Times it.

TEACHER: By, by using times? Okay. Uh-huh.

EAGLE: How you get 4 to 8 times 2.

TEACHER: Okay.

EAGLE: And then I added the same thing to the top. May I put 6 on top because the top was 3 times 2 was 6.

TEACHER: Umhmm.

EAGLE: The bottom one is the denominator, are the same. So, it was just times one, and then you do the times one on the top—

TEACHER: Okay

EAGLE: --because the top one is times one, too.

TEACHER: Umhmm.

EAGLE: I just put the one here.

TEACHER: How do you know to use the same number for the top and the bottom?

EAGLE: Huh?

TEACHER: How—Why did you times by the same number on the top and the bottom by two and two here and one and one there? Why?

EAGLE: Cuz you have to do the same that you do with the bottom.

TEACHER: Oh, oh, okay. So what did you make when you have $6/8$ ths?

EAGLE: What do I make?

TEACHER: Yup. So $3/4$ ths becomes $6/8$ ths. How come we can use $6/8$ ths and not $3/4$ ths?

EAGLE: Well, because, this kind of hard to do differently now, adding the [*inaudible*]

TEACHER: Oh, okay! Okay! Cool. Awesome. Can you—so you had your two fractions here and now what do you do?

EAGLE: And now the denominators are the same now— [*Step 3*]

TEACHER: Okay.

EAGLE: --and you just add six eight plus $1/8$ th—

TEACHER: Umhmm.

EAGLE: --and you just have to add the 6 and 1.

TEACHER: Umhmm.

EAGLE: For seven eight you just put the 8 on the bottom.

TEACHER: Okay.

EAGLE: Then, that's right.

[Steps 3 and 4 throughout]

TEACHER: Great. Alright! So there's just one more problem. I'll take that one from you. Do the same thing: You're gonna read it, then you can just solve it with the math. You don't have to write out complete sentences if you don't want to. You can just show me what you do for math class. Okay? And, then you're going to explain it to me after you're done. *[student nods]*

EAGLE: So can I do it just the numbers?

TEACHER: Hm? Yes, yes, yes.

EAGLE: *[looks up]*

TEACHER: Okay, can you tell me what you did? With your work?

EAGLE: Find the LCM first. *[Step 3]* Six, and have $2\frac{1}{4}$ th and four, three, and eight. Then, you have to find it. *[Step 4]*

TEACHER: Okay. So, how did you do that? How did you find the LCM?

EAGLE: First, ignore the whole numbers / the whole numbers. Just the fractions, so that's, so that will be like the LCM for 8 is 8, 8, 16, 24. For 2, is 2, 4, 6, 8. Find for 8 is 8, 16, 24. For the other one, 3 and $\frac{1}{8}$ th. *[Step 4]*

TEACHER: Okay.

EAGLE: So, you look for the LCM, and the LCM is 8.

TEACHER: Umhmm.

EAGLE: Put the 8 under the fraction like this *[shows teacher]*. So, you write the whole number—

TEACHER: Umhmm.

EAGLE: --and add your whole numbers. You just put it there, like that.

TEACHER: Okay, so.

EAGLE: You put the 8 in every number—

TEACHER: Okay, so that's what you want your denominator to be, gotcha.

EAGLE: You put it under here and then, um, on the top just 2 and $\frac{1}{4}$ th. And getting from 4 to 8 is timesing 2.

TEACHER: Umhmm.

EAGLE: Then from 1 times 2 equals 2.

TEACHER: Okay.

EAGLE: Then I did the same thing to 6 and $\frac{1}{2}$. How do I get from 2 to 8? Times 4.

TEACHER: Umhmm.

EAGLE: 1 times 4 equals 4. And 3, 3 and $\frac{1}{8}$ th, how do I get from 8 to 8? It's just timesing 1. So, 1 times 1 is 1.

TEACHER: Umhmm.

EAGLE: And then, I add all of them up. So, like the 4, 1, and 2.

TEACHER: Okay.

EAGLE: And then I get 7. And then the whole numbers, you add them up, like 6, 7, 8, 9, 10, 11.

TEACHER: Okay.

EAGLE: And then you just put it right here.

TEACHER: Okay. So how come the whole number is separate from the fraction? How did you know to do that?

EAGLE: Because there's like, um, 11, like one big circle that's a fraction. There's like one whole pizza, so instead of that, there's 11 pizzas. *[Step 6]*

TEACHER: Eleven pizzas, okay. And then what is the fraction?

EAGLE: The fraction is like, um, one pizza. There is $\frac{7}{8}$ ths of it.

TEACHER: $\frac{7}{8}$ ths of it. Okay. Cool! And then, um, the other question I was gonna ask you actually about this one and this one is how do you know to add on these? What words tell you that you need to add? What—

EAGLE: Like, on this one, it says “How many miles did Evelyn need ride in the, in the three days, all put together.” Only she ride three days. *[Step 1]*

TEACHER: Okay. And then, how about for this one about the pizzas?

EAGLE: The pizza, it says that, um, “How much of the whole pizza do you eat altogether?” Like, “altogether” means adding. *[Step 1]*

TEACHER: Okay. So you knew that that word meant adding? Alright! Awesome! Thank you very much, Eagle!

Ariana’s Recording:

TEACHER: Okay, Ariana, go ahead and read this problem, and try and solve it. Try and write down work if you need to, and then once you’re ready, I’ll ask you to tell me what you’re thinking. Okay? Whenever you’re ready.

ARIANA: *[Reading problem]* Suppose you ate $\frac{3}{4}$ ths of a pizza, and then you ate $\frac{1}{8}$ th of another pizza. And the same size. How much of the whole pizza did you eat altogether? I don’t—

TEACHER: Well, if you want to write down and do some math first, that’s fine. And then you can tell me what you’re thinking.

TEACHER: Okay. Are you done? Do you need—

ARIANA: Done.

TEACHER: Okay. Alright, can you tell me what you did?

ARIANA: I drew a circle/, and it had three pieces in it. *[Step 3]* And I filled in one,/ one of the pieces,/ and then I put $\frac{1}{8}$ th. Then I drew another circle,/ and I drew 4 pieces in it,/and I colored in $\frac{3}{4}$ of them, and that equal $\frac{3}{4}$ ths. And then,/ so I know then the shaded pieces are 4 then. And the remain/ing ones,/ there’s/ 6, so it’s four sixths. *[Step 4]*

TEACHER: There are four six—okay, so there’s six pieces left of the ones that aren’t colored in is what you’re saying in both the circles together.

ARIANA: Yes.

TEACHER: Okay. Um, so is there anything else, any other ways that you think you could maybe could figure this out?

ARIANA: Like subtracting them or something like that?

TEACHER: I dunno, just what do you—I mean, if you don't think so that's fine. Good. Is there any other way you can think of that somebody else might solve the problem?

ARIANA: No.

TEACHER: No? Okay! Awesome! Let's try just one more, Okay? So, again, you can read it, you don't have to read it out loud if you don't want to. If you just want to read it to yourself that's fine. And then, you just go ahead and solve down here, and then tell me what you're thinking. Okay? Go ahead.

TEACHER: Okay? So what were you thinking?

ARIANA: Um, right away I noted that the denominators are different, so, um, I know that I had to change to a denominator that would be equal at the bottom. And there's, and there's a fraction that says 6 and $\frac{1}{2}$. [Steps 1,2 and 3] Six over one. Six over one, two. And I know that, and I know that right away that I saw that, that I, that that could stay, the eight could stay like that, on this— [Step 4]

TEACHER: The 8 in the denominator.

ARIANA: Yeah. Yeah, so I, so I, so one of the problems up here, um are 3 and $\frac{1}{8}$ th. And so that one could stay the same, so like, there's another one, 2 and $\frac{1}{4}$ th, that, um, that can change into 8, by the fourth timesing, by, times, times 2. And so whatever times by at the bottom you have to times at the top. And then, on the, on the first one you have to times it by five—four. And four at the same—at the top two. So, the new fractions would be 6 $\frac{4}{8}$ ths, 2, $\frac{2}{8}$ ths, 3 $\frac{1}{8}$ th, eleven—Oh! At the end, all those fractions would equal 11 and $\frac{7}{8}$ ths. [Step 4]

TEACHER: Um-kay. And then what is this part on the bottom here?

ARIANA: At the bottom I had to tried, um, I tried, I had to try, um, the multiples cuz I didn't know what I could times the 2 by. So, I had to, um, count by 2s: 2, 4, 6, 8, 10, and 12, so I knew 8 was in it, so I knew I could change the denominator to 8 as well. [Step 3]

TEACHER: I see. Okay. Cool. So this is kind of your plan, you got to use the LCM first before everything else. Cool! So, how could you, like, check to make sure your answers are right? Do you have any ideas?

ARIANA: Ummm / Uh, K / Like, list all the multiples, like make sure they all have 8s in them? *[Step 6]*

TEACHER: Okay! For each, each of these top fractions here?

ARIANA: Yeah.

TEACHER: For the denominators, that's what you would use?

ARIANA: Uh-huh.

TEACHER: Cool! Awesome job! That looks really good! Thanks!

[Steps 1,2,3,4 and 6 used]

Esmeralda's Recording:

TEACHER: Ok, so, Esmeralda, we're going to have you read a problem, and we're gonna have you think about the steps that you'd use to solve it, and then you can write them down, anything that you're thinking, and then after you're done with your writing you can tell me out loud what were the steps that you used to solve the problem. K? We'll let you start.

ESMERALDA: So I draw any pictures?

TEACHER: You can do whatever. If you draw pictures normally that's great; if you do something else, that's great, too!

ESMERALDA: There.

TEACHER: Okay. Okay, could you explain to me what you're thinking, can you walk me through your steps, here? Just tell me what you did first, second, third, then what did you do?

ESMERALDA: I read it, then, umm / Drew picture, *[Step 3]* and then I did $3/4$ s, and then $1/8^{\text{th}}$, and then I did the LCM for it, *[Step 3]* of number 4 and 8, and then I *[pencil taps]* /

TEACHER: Which, this one right here?

ESMERALDA: Yeah, and then I did that.

TEACHER: Okay. Can you just tell me what, like, what math you did?

ESMERALDA: I did 8 times 1, which is 8, then 1 times 1, and that equals $\frac{1}{8}$ — [Step 4]

TEACHER: Uh-huh.

ESMERALDA: --and then I did four times two, which is 8, and then I did 3 times 2, which is $\frac{6}{8}$ ths, and then I got $\frac{7}{8}$ ths. [Step 4]

TEACHER: Okay. How do you know how to add those two fractions?

ESMERALDA: Cuz there's how much all together. [Steps 1 and 3]

TEACHER: Okay. So, the word, this word, “all together”, is that, is that what told you to add? [student nods][step 1]You knew that means add? Okay. Cool. And, so, how did you know to multiply them to get this 8 here, for the bottom here?

ESMERALDA: Cuz they were different denominators, and—[Step 2]

TEACHER: Oh!

ESMERALDA: Yeah!

TEACHER: Okay. So, then, when they're different denominators, what do you do?

ESMERALDA: You do the LCM—[Step 3]

TEACHER: Okay.

ESMERALDA: --and when they're the same you don't.

TEACHER: Okay. And, what did these pictures do for you? How come you drew those pictures?

ESMERALDA: I dunno.

TEACHER: Okay, well that's great! Thank you very much, Esmeralda! I just have one more for you. So we're gonna do the same thing, you're just gonna read it, and read it to yourself if you'd like, and then you're gonna, if you would like to, write down any of these steps that you used to solve the problem. And then you're gonna tell me out loud what you were thinking. Okay?

ESMERALDA: Yeah.

TEACHER: Alright, go ahead.

ESMERALDA: *[whispers]* This one's hard!

TEACHER: Just do the best you can. Whatever you can think about when you're doing it.

ESMERALDA: Okay, I'm done.

TEACHER: Okay, can you think about anything else that you would need to do to solve the problem?

ESMERALDA: Times it? Times them?

TEACHER: Okay, so, go ahead and write down what you're thinkin', and then you can tell me out loud. When you need times, go ahead and write it down, then tell me what you thought.

ESMERALDA: Okay. Okay, I did my best.

TEACHER: Okay, great! Can you walk me through it, tell me what you did? So first, what happened?

ESMERALDA: I wrote the numbers—

TEACHER: Umhmm.

ESMERALDA: --then I drew a picture of the fraction, and then I did the LCM for 2, 4, and 8, and I got 8 for all of them. And then I wrote that, um, fractions again, but I wrote 8 instead of 2, 4, and 8. *[Steps 3 and 4]* So I wrote all 8s, and then I got 11 and $\frac{3}{8}$ ths. *[Step 4]*

TEACHER: Okay, um, and when you got stuck up here, and you said, "This one's harder, I don't know what to do next". How did you come up with "times it"? Oh you said, "I've gotta times them". And then you wrote the LCM. How did you get from here to there? What were you thinking in your head?

ESMERALDA: That these two numbers weren't the same, so, and then, like, I came up with doing the LCM. *[Step 3]*

TEACHER: Umhmm.

ESMERALDA: And then, yeah.

TEACHER: Okay, so you saw that the three bottom numbers weren't the same, and so then that made you think "Oh, I have to find the LCM"? What does the LCM tell us, again?

ESMERALDA: That it's 8.

TEACHER: Okay, that what's 8? / Well, you wrote it here, what did you do here?

ESMERALDA: I put the 8s, in / on the bottom /

TEACHER: Okay.

ESMERALDA: / of the fraction.

TEACHER: Oh, okay, cool. Do you remember what that bottom number is called, in a fraction?

ESMERALDA: Denominator?

TEACHER: Okay. So the denominators, you thought, should all be 8, because it all seems 8. Okay, and then how did you get the $\frac{3}{8}$ ths, again?

ESMERALDA: Because I added the ones.

TEACHER: Okay, good. So, and what are the ones?

ESMERALDA: The nu-rum-inator?

TEACHER: Yeah, right, the numerator, yup. Okay, cool. And then where did this 11 come from?

ESMERALDA: Adding this two numbers.

TEACHER: Okay. Awesome! Thank you very much, Esmeralda! That was great!

[Steps 1, 2,3 and 4 used]

Melissa's Recording:

TEACHER: Okay, so. Go ahead and read it through, and then, just go ahead and write down whatever you're thinking, and then I'll ask you to explain it in just a minute. Okay? Go ahead.

TEACHER: Are you done? Okay! Can you ex / walk me through it, step by step? What did you do?

MELISSA: Okay. First, I read like the steps, then I put number one's LCM. Then I just put like the LCM, 4, and then, then, the I just like, tried to multiply. Like 4 times 2 equals 8. And then, um, and then 8, and then the, and then the same number is 8, and then I multiply 3 times 8 which equals 24, and 4 times 8 equals 32. Then, 1 times 8 equals 8 and 8 times 8 equals 64. Then I add 24 out of 32, and 8 out of 64 equals 32 out of 96. *[Steps 3 and 4]*

TEACHER: Okay. And how did you know to find the LCM at the beginning?

MELISSA: Because always the, the bottom numbers need to see if it fits on the other number. *[Step 3]*

TEACHER: Okay. And how did you know to add the fractions together?

MELISSA: Because, if you have, like, put in 4 plus 8, it will give you the number that equals the same. And then you find, like, you add the bottom number, and then it will give you another. *[Step 3]*

TEACHER: Okay. And were there any words in the question that helped you know that you needed to add?

MELISSA: Yeah.

TEACHER: What words?

MELISSA: "How much". *[Steps 1 and 3]*

TEACHER: "How much"? Anything else?

MELISSA: Yeah. And "altogether". *[Steps 1 and 3]*

TEACHER: Okay, yeah, nice. I see that you underlined that three times. *[Step 2]*

MELISSA: Yeah.

TEACHER: You thought that word was important, was that what that meant?

MELISSA: Yeah.

TEACHER: Okay. Cool! Thanks! Okay, and then the second problem, you're gonna do the same thing. Go ahead and read it, decide what your steps are gonna be, and explain it to me.

MELISSA: Okay.

MELISSA: Okay.

TEACHER: Okay! Can you explain to me what you did?

MELISSA: I, like, I found another one, like the MC / LCM. And then, I just put, like, the 3, the, the bottom numbers, the 3 of them. Then I put 2, 4, and 8. Then I guess 2, 4, 6, 8. *[Steps 3 and 4]*

TEACHER: Umhmm.

MELISSA: Then on the other ones, 6, 8. So then, so then I have 6 and $\frac{1}{2}$. And then I have 6. The 6 stay the same, and then I put a one and an 8 on the bottom. Then the 2 and the $\frac{1}{8}$, 3 and $\frac{1}{8}$. Then, I just add them, and the other numbers stay the same. Then I just add up the big ones, and they give me 12 and $\frac{1}{8}$ th. *[Steps 3 and 4]*

TEACHER: Okay. Great. And, how, about, how did you know to add these?

MELISSA: Said "How many miles did Evelyn ride in the three days?" *[Step 1]*

TEACHER: Okay. And so, you underlined 3 days 3 times. How come?

MELISSA: Because 3 days, is, they were saying like the 3 days like, um, like on Monday, Tuesday, and Saturday, they were the three days. *[Step 2]*

TEACHER: Ah! Okay. So the three numbers, you're saying, you knew you needed to put them together.

MELISSA: Yeah.

TEACHER: Okay. Gotcha. Okay, great! Thank you Melissa!

[Steps 1,2,3 and 4 used]

Apple's Recording:

TEACHER: Okay, So. Okay, so first I'd like you to read the problem. You can read to yourself or out loud if you want, and decide what you're gonna do. Write down your steps, and then you're going to explain it to me out loud. Okay? Go ahead.

APPLE: *[Reading the question]* Suppose that you eat $\frac{3}{4}$ of a pizza, and then you eat $\frac{1}{8}$ th of another pizza of the same size. How much of a whole pizza do you eat altogether?

TEACHER: And you don't have to write all the sentences out if you don't want. You can just write what you're thinking down.

[1 min passes]

APPLE: K.

TEACHER: And, can you actually solve it, solve the math, too? Go ahead and follow your steps and solve it.

APPLE: K.

[1 min passes]

TEACHER: Okay? Can you walk me through it, tell me what to do?

APPLE: First, I do the LCM, then I did the number. Put it on the top, so, wait, I mean on the bottom. Uh, Uh, I— *[Step 3]*

TEACHER: So what did you do here, then?

APPLE: I, I put the number of the LCM back on, uh, and I put on the bottom the 8 and the 8. Then I saw that 8 times 1 equals 8, so I did the same on the top. *[Steps 3 and 4]*

TEACHER: Umhmm.

APPLE: And, and I got 1. Then on here I did 4 times 1 gets me to 8, uh, it's 2. Then I did 3 times 2, which equals 6. Then I added the top numbers, and the bottom number stays the same. *[Steps 3 and 4]*

TEACHER: Great. And, what about this problem told you that you needed to add?

APPLE: Hum? Oh!

TEACHER: Were there any words or anything that tipped you off?

APPLE: How much, uh, "altogether"? *[Step 1]*

TEACHER: Ahh! So you saw that word and you saw that was add?

APPLE: Uh-huh I did.

TEACHER: Okay. Great! K, so you just have one more. You're gonna do the same thing. Just read it, and then you can just go ahead and solve it, do all your math steps, everything, and then explain it to me. Okay?

APPLE: Alright. *[Reading the question]* Eleven rode her bike 6 and $\frac{1}{2}$ miles on one day. On Wednesday she rode 2 and $\frac{1}{4}$ miles. On Saturday, she rode 3 and $\frac{1}{8}$ th miles. How many miles did Evelyn ride in three days? Do I write the steps, too?

TEACHER: Uh huh! Yup! Everything that you're thinking, just write it down.

[1 min passes]

TEACHER: Okay? So go ahead and do the math and solve it.

[30 sec pass]

TEACHER: You can do it however you want. Yup, whatever you think.

[30 sec pass]

APPLE: I don't know what that 7 means.

TEACHER: What's that?

APPLE: I don't *[inaudible]*

TEACHER: What do you think? Do what you think, and then tell me what.

[1 min passes]

TEACHER: Ready to explain it to me?

APPLE: I think so.

TEACHER: Alright! Show me what you did! Apple, what did you do?

APPLE: First, I add the fractions and I see if I can do LCM. So, I do the LCM and I do 2 and 4 and 8. *[Step 3]*

TEACHER: Umhmm.

APPLE: Then I solve the problem. Then I put the equal there. Then I put 8 on the bottom. Then I solve the four—two times, 2 times, 2 times 4 equal 8, so I did 1 times 4, which equals 4. [*Steps 3 and 4*]

TEACHER: Umhmm.

APPLE: And down here I did uh, 4 times 8—4 times 2 equals 8. So I did 1 times 2, which equals 2. [*Steps 3 and 4*]

TEACHER: Umhmm.

APPLE: Then I did 8 times 8—8 times 1 which equals 8. And, um, 1 times 1, which equals 1. [*Steps 3 and 4*]

TEACHER: Okay. So then...

APPLE: And then I did the top numbers. I put the 7, then I put the 8 on the bottom, which stays the same. [*Steps 3 and 4*]

TEACHER: Okay! Awesome! What were you doing over here with this picture here?

APPLE: I was trying to do, like, a pie thing. [*Step 6*]

TEACHER: Umhmm.

APPLE: So, we'll see. I did a half—

TEACHER: Umhmm.

APPLE: --a half here, and a $1/4^{\text{th}}$ here. And I saw if I could do $1/8^{\text{th}}$. [*Step 6*]

TEACHER: Umhmm.

APPLE: But I couldn't. I think / I don't know what that could be.

TEACHER: Oh, okay. So, you were trying to look at it as a picture, and split it into / to see if that worked.

APPLE: Yeah. See if I could add up the two--

TEACHER: Into a whole circle? Or—

APPLE: --yeah. I think that I forgot to add these. [*Step 5*]

TEACHER: Oh okay! Okay. So this, you were trying to see if you could make it add up to a whole circle, is that what you were trying to do?

APPLE: Yeah.

TEACHER: Oh, okay. Gotcha. And so you saw the denominator, so that's what these pieces are?

APPLE: Yeah.

TEACHER: Okay! Cool! Thank you very much, that's awesome!

[Steps 1, 2, 3, 4, 5, and 6 used]

Round 2-March

Maria's Recording:

Teacher: Okay, so, the first question is here. I want you to read it just like we did last time, um and you're going to decide what steps you would do to solve the problem and to find the answer and when you're done, you're gonna just say, okay, I'm ready to explain and you're gonna explain your thinking out loud. Okay? Go ahead and start.

Student: What's a half plus a half? Oh, one whole.

Teacher: Yep, good.

Student: Okay. I'm done.

Teacher: Okay, alright, tell me what you were thinking.

Student: Um, I was thinking that how many sixes go into 65.

Teacher: Okay and where'd you get that 6 from?

Student: From the $6\frac{1}{2}$.

Teacher: How did you know what to do with those numbers?

Student: Cuz the perimeter needs to equal 65 square meters. In the question. *[Steps 1 and 2]* So then I said how many sixes go into 65 and I said 10 and then I said how many half? And I said one so 61. The 60 comes from the 10 plus 6 but then the two halves equal one whole. *[Steps 3 and 4]*

Teacher: And so this 61 is the answer to what?

Student: the answer to the perimeter.

Teacher: Anything else that you want to tell me about the problem?

Student: NO

[Steps 1, 2, 3, and 4 used]

Teacher: Okay, awesome so we just have one more. So it's the same kind of thing as before so we'll just have you read through it, write down any steps that you took and then we'll have you explain out loud what you were thinking. Okay, go ahead.

Student: Done.

Teacher: Okay, so what's the first thing you thought of when you read the problem?

Student: That the area has to equal like 24, uh, 240 cuz it says that in the problem. *[Steps 1 and 3]*

Teacher: Okay, what else?

Student: and then 6 times 4 equals 24 so I said 60 times 4 equals 240 and then I drew the design rectangle that has a length of 4 and then a width of 60. *[Step 4]*

Teacher: How did you know to use times? With a length of 4 and a width of 60?

Student: because in math they say that the length times the width equals the area. *[Step 3]*

Teacher: Okay, then what?

Student: Then I did 40 times 6 and 30 times 8 cuz 4 times 6 and 8 times 3 equal 24.

Teacher: You have four lengths and widths here. How did you know to find all of them?

Student: Well cuz the problem says to write down all the numbers you know. So I was like thinking of what gives me 24 and then I added zeros on. *[Steps 1 and 3]*

Teacher: Is this all of the possibilities?

Student: No, I think there's more but like I don't know all of them.

Teacher: Is there a way you could check?

Student: Yeah, I can divide with the calculator. So 2 times 12 is 24 so 20 times 12 is 240.

Teacher: How did you know to use division in your calculator?

Student: Cuz division is the opposite of the times so you take the answer when you divide. *[Step 6]*

Teacher: Gotcha. Great. Thanks.

[Steps 1, 3, 4, and 6 used]

Eagle's Recording:

Teacher: What you're going to do is read the problem. You may read to yourself and then decide what you need to do for the problem. Do all your work and find your answer and then when you are ready you can say I'm done and explain to me out loud what you were thinking each step of the way. Okay, ready?

Student: Yes. Done.

Teacher: Okay, walk we through it. What did you do?

Student: For the first one I did 240 divided by four because on the whole area she has 240 square feet. *[Steps 2 and 3]* She wanted to make a rectangle floor *[Step 2]* so I divided 24 by four and I did four because there's four sides on a rectangle *[Step 3]*. When I divided I got 60 and I put 60 on all the sides so when you add all of them up they would equal 240. After that I did 240 divided by three and I got 80. For this one I just guessed 110 and 110 and I add them to get 200 so then I know to use 10 and 10 on the other sides so it equals 240. *[Steps 3 and 4]*

Teacher: How did you know that in the problem you needed all these rectangles?

Student: It says to list all the possible numbers. *[Step 1]* I did some of them but there might be more. *[Step 5]*

Teacher: Okay, great. We have one more problem and I want you to do the same thing. Read the problem, write down any steps you took to solve and when you are ready, go ahead and explain it out loud to me. Go ahead and begin.

Student: Done. First it says the rectangle floor area of the storm shelter is 65 square meters and its length is six and a half meters. What is the width of the storm shelter? What is the perimeter? *[Step 1]* So I did 65 divided by six and a half and then I got ten. *[Step 4]* I tried to divide it because it might be the other half of the width cuz 65 is the

whole thing *[Step 2]* and if I divide it it might give me like the other side. *[Step 5]* Then I drew a rectangle and I put six and a half on one side and ten on the other side. *[Step 3]* Then I added all of them up and I got 33 cuz perimeter is adding so the perimeter is 33. *[Steps 3 and 4]*

Then I check in the calculator by putting in 6.5 times ten and I get 65 so I know my answer is right. *[Steps 5 and 6]*

[Steps 1, 2, 3, 4, 5, and 6 used]

Ariana's Recording:

Teacher: Okay, so I am going to give you a problem, just like last time and you are going to read and figure out what you need to do and solve it and write it down if you like. Then once you are satisfied with your answer, you can say okay, I'm ready and I'll ask you to explain your thinking out loud. When you're ready, you can go ahead and start.

Student: I'm ready. Okay, when they said list all the possible lengths and widths that Alyssa could make, I thought of ten because it's like an easy number. *[Step 1]* I decided to try 240 times ten and I got 2,400. I got the 24 cuz the problem said there is a rectangular area of 240 square feet. Oh, wait, um, I think I did it wrong. *[Steps 3 and 4]*

Teacher: So tell me what you are thinking.

Student: I think I was supposed to do 240 divided by two to get 120 and 120 or something. SO I am thinking that since it says that she wants to make it 240 and no bigger that I should divide instead. *[Step 5]* At read list all the possible lengths and so I was thinking multiply by ten, then twenty and then fifteen. *[Step 3]*

Teacher: So were you thinking that 240 was one of the lengths?

Student: Yeah.

Teacher: Are there any other lengths that you think you can get?

Student: You can do 240 divided by 3 and you get 80 cuz all the possibles means more than one. *[Steps 1 and 4]*

Teacher: How did you know what to do with the lengths and widths?

Student: Cuz it says the area and I know that area equals length times width. *[Steps 1 and 2]*

Teacher: Okay, anything else you want to tell me?

Student: No

Teacher: Okay, so the second one is really similar. You're going to read it, write down your steps, tell me step by step what your thinking and say it out loud. Okay? You may begin.

Student: Done.

Teacher: Okay, tell me what you did.

Student: It said in the question what is the perimeter and what is the width of the storm shelter and um, I knew right away that the width meant the area, well not the area but the space around it / and so the perimeter is like when you add up all the sides and so it said that the storm shelter was 65 square meters and the length was six and a half meters and *[Steps 1, 2, and 3]* so I knew right away that I had to find the perimeter which you have to add all the sides and I did 65 plus 65 and that would equal 143, the perimeter. *[Step 4]* For the area I did length times width so six and a half is the length and 65 square meters is the width. *[Steps 3 and 4]* Wait, doesn't it go length times width divided by two or is it base times height? *[Step 5]*

Teacher: Well, what do you think? What kind of shape are we dealing with?

Student: Rectangular, so I think length times width?

Teacher: Right, you did the right formula.

[Steps 1, 2, 3, 4 and 5 used]

Esmeralda's Recording:

Teacher: Okay, so I am going to give you a problem and this one has to do with area and perimeter and I am going to ask you to read it and then after you read it, you can write down on the paper what steps you did and then if you are finished you can just say, okay, I'm done, I got it all, I'm ready to explain by answer and you just walk me through each step out loud. Okay? Go ahead.

Student: I don't know. Can I try like 100 and 100?

Teacher: Sure, if you use that as your strategy, you can try it.

Student: Done.

Teacher: Okay, what does that tell you?

Student: cuz it equals up to 240. Cuz 20 plus 20 equals 40 and 100 plus 100 equal 200 so together it is 240. Also it said to use the whole numbers in your answer. *[Steps 1, 3, and 4]*

Teacher: Is there anything else you want to tell me about the problem?

Student: No.

Teacher: Okay so we will try one more. So go ahead and read it, write down the steps you are thinking in your head and then tell me what you are thinking out loud. Okay? Try this one.

Student: I got 1982.5 in my calculator. *[Step 4]*

Teacher: Okay and what does that tell you?

Student: The area? *[Step 1]*

Teacher: you think that is the area? Okay.

Student: Then it asks me for the perimeter. *[Step 1]* So I think you divide 1982.5 by two. *[Steps 3 and 4]*

Teacher: Why do you think that?

Student: cuz we do the base times the height divided by two to get the perimeter. *[Step 3]*

Teacher: Anything else?

Student: the problem says we need to find the width. *[Step 1]* I think that's six and a half cuz it says the length is six and a half and that is like the same as the width. *[Steps 2 and 3]*

Teacher: Anything else?

Student: Nope.

[Steps 1, 2, 3, and 4 used]

Melissa's Recording:

Teacher: Okay so, I'm gonna give you two problems and the first problem is hear. You're gonna read the problem and then you can use the paper to write down any steps that you use to solve it and then as you as you are ready and you have your answer, you can explain it to me out loud. Okay, go ahead.

Student: Ready.

Teacher: Okay, so talk me through it.

Student: So first I said 240 divided by something and I thought twelve and then I said twelve times ten equals 120 [Steps 3 and 4] and then I thought that's not even close [Step 5] so I put 12 times 20 equals 240 and that how I get 20 feet on the length and on the width I get 12 feet. I checked by multiplying 20 times 12 and I got 240 feet squared so it was good. [Step 6] Then I knew I had to find the perimeter [Step 1] which is 64 feet cuz I add all of them.

Teacher: Can you back up to the beginning of the problem when you drew the rectangle and figured out how to use 240.

Student: Cuz it said the floor area of 240 square feet [Step 2] and I underlined it and then it said length and width [Step 1] and I know we have to multiply length times width cuz that's how we get area and we add all the sides and that's how we get the perimeter (step 3) and I knew that the length on the bottom and the top in my drawing have to be the same because rectangles have equal top and bottom. [Step 3]

Teacher: Great. Anything else for that problem?

Student: No

Teacher: Okay, in the second one you're gonna do the same thing. You are going to read it, figure out your work down here was you solved it and you are ready to explain and you just say out loud what you did. Go ahead.

Student: Okay, ready.

Teacher: Go ahead.

Student: So again it said rectangular and then it said its length is six and a half meters and it said what is the width gonna be and then what is the perimeter [Steps 1 and 2] So I did two eighths and two eighths because 65 divided by six and a half is two eighths and then to get the perimeter I did two eighths plus two eighths and I got one. [Steps 3 and 4]

Teacher: Okay, I see that you changed six and a half to thirteen over two. Why did you do that?

Student: Cuz whenever we don't have a whole number if we don't know how to do it, we can change it to an improper fraction but I decided to go back to the six and a half. *[Step 6]*

Teacher: Okay, thanks.

[Steps 1, 2, 3, 4, 5, and 6 used]

Apple's Recording:

Teacher: Okay, so I'm going to give you one word problem and I want you to go ahead and read it and decide what kind of steps you would use to solve it and then you can write it down if you would like to and then after you've thought through it and you know what your answer is gonna be then you're gonna talk me through what you were thinking. Okay? Go ahead.

Student: Okay, I'm ready.

Teacher: Okay, explain it to me.

Student: I did 240 divided by two which gave me 120 and I put the lengths and widths there on my drawing so if you do 120 times two you get 240. *[Steps 3 and 4]* I knew to do that cuz it says 240 square meters *[Step 2]* and it says list the possible lengths and widths. *[Step 1]* I also know that area equals length times width and it says area here. *[Steps 1 and 3]* Then I did 240 divided by 3 and I kept going bigger and I got 80 and 240 divided by 80 equals 3 and like 60 times four and 48 times five. *[Step 3]*

Teacher: How do you know that you're done?

Student: Cuz right here I did 240 divided by 7 and it gave me decimals and it says I only write the whole numbers. I went in order so I wouldn't miss one. *[Steps 5 and 6]*

Teacher: When you first read this problem, what did you think about?

Student: I made a plan to divide cuz when you divide 240 by 2 you get 120 and that's the length and the width. I needed the length and the width. *[Steps 3 and 4]*

Teacher: Thanks. Is that everything?

Student: Yep.

Teacher: Great. In the next one, I want you to do the same thing. I want you to read it, show any work that you want to and explain out loud to me at the end. Okay? Go ahead.

Student: I'm done.

Teacher: Okay, so tell me. What's the very first thing you thought of when you read the problem?

Student: First I thought to put 6.5 because six and a half is the same as 6.5 in a decimal. *[Step 3]* Then I tried to find 6.5 times what could get me to 65. *[Step 3]* So I did 65 divided by 6.5 and I got 10 and I checked 10 times 6.5 which got me 65 which I thought would be the area cuz 10 and 6.5 make like a fact family with 65 *[Steps 3, 5, and 6]* Then for the perimeter I did six plus six and a half plus a half and I got 13 and I put that for the two lengths and to figure out the width I thought ten plus ten equals 20 so if you add them all up you get 33 which is the perimeter. *[Steps 1, 2, 3, and 4]*

So like I thought first to do the width cuz I know the length and the area and then I needed the perimeter. *[Steps 2 and 3]*

[Steps 1, 2, 3, 4, 5, and 6 used]

Round 3-May

Maria's Recording:

Teacher: The first problem is right there and I would like you to read it, find any key words, use any strategies or any steps that you'd like to solve it. Write it down and then explain it to me out loud. Go ahead.

Student: okay. 3.39 cuz I wrote the numbers in order from least to greatest and them crossed out on each side until I got to the middle *[Step 3]*. Part b I added all them up and I got a number and then I divided it by five cuz that how many numbers there are cuz that's what you do for the average. *[Step 3]* For part c, I was tryin to find the outlier in the data and it's 3.89 because I think that if some of the numbers are the same and only one is different, then it's an outlier. *[Steps 3 and 4]*

Teacher: What do you mean by the same?

Student: If they are the same I mean they are in the same range like close together, then that's not the outlier and 3.89 is far away so it is the outlier. *[Step 1]*

Teacher: Got it, thanks. One more of the same thing. Read it. Make sure you tell me any of the key words or phrases that you pulled out. Write your steps down and then explain.

Student: Okay ready. For part a, I added all of the numbers together and divided it by five because I have to find the average [*Steps 1, 2, and 3*] and I got 8.72. [*Step 4*] Then it said what is the median [*Step 1*] and I did the same thing. I put them in order and crossed out to the middle [*Step 3*] and I got 68 [*Step 4*]. I used the low numbers cuz it says to use the low temperatures [*Step 1*]. For the last one it said the mode and I think the mode [*Step 1*] means, like, the most so I put 79 because I see it three times [*Step 2*] and none of the others have more than one of the same number.

Teacher: Awesome. Anything more you want to tell me about these?

Student: Nope.

[Steps 1,2,3, and 4 used]

Eagle's Recording:

Teacher: Okay, so, um, just like we've done before in the past, we've talked about word problems, right? And you've been doing different ones, so this time you're gonna have one that had to do, actually both of them have to do with mean, median mode, range... that stuff that we've been working on, okay? So you're gonna read the problems and you're going to decide what steps you're gonna take, you're gonna write the steps down and then once you're finished writing and you're ready to explain it to me, then you can stop and say okay, and then explain it all, whatever you were thinking / and you may use a calculator and here is question one. You may begin.

Student: Isn't average the most?

Teacher: Ah, do you remember another word we know that has to do with average?

Student: Most?

Teacher: If that's what you think, then try it.

Student: I'm done.

Teacher: Okay, go ahead and explain it to me. What's the first thing you did?

Student: First, it said what's the relay team median practice time. [*Step 1*] I got 3.39. [*Step 4*]

Teacher: Okay, and how did you get that?

Student: I saw median and I know middle. [*Step 3*] I put the numbers in order from least to large and I crossed out one from each side. [*Step 3*]

Teacher: What else did you do?

Student: I read what is the relay team average time and I thought average means bigger number [Steps 1 and 3] and so I saw 3.89. [Step 4]

Teacher: Okay, anything else on that one?

Student: Then it said is there an outlier in the data and I said 3.89 because all the numbers, the rest of the numbers are under .50 but 3.89 is like the way high number. There's nothing close to it. Outlier means the number that is far away from the rest. [Steps 1, 2, 3, and 4]

Teacher: Could the outlier be smaller than the other numbers?

Student: Yeah, it could've been .14 but there's .24 too so it has to be the bigger number since it's farther away [Step 5]

Teacher: Awesome, great, so you're gonna do one more and you're gonna do the same thing... think about the steps you're gonna use, map it all out for yourself, write down and then explain it, okay? Go ahead.

Student: I'm done.

Teacher: Done? Okay, can you explain to me, what's the first step you did?

Student: It says what was the mean (average) high temperature for the five days and I said 93 cuz that's the highest temperature in the high [Steps 1 and 2]

Teacher: and how did you know to use the high temperatures?

Student: Because it says average high temperature in the question and I saw average so I looked for the highest cuz that means the most [Steps 1, 2, and 3]

Teacher: anything else?

Student: Um, it says what was the median low temperature for the five days and I lined them up again from least to greatest and I crossed the numbers out cuz median means the middle number and it is in the question [Steps 1, 2, 3, and 4]

Teacher: Okay and then?

Student: What was the mode for the 10 temperatures? It was 79 cuz there's three 79s and mode means most often. [Steps 1, 2, 3, and 4]

Teacher: Great, thank you very much!

[Steps 1,2,3,4, and 5 used]

Ariana's Recording:

Teacher: Okay, so, we have two problems and I would like you to start with this one. Go ahead and read it all and decide what you are going to do. Write down any steps that you need to to organize your thoughts and then you can say, okay, I'm ready and you will talk to me about it out loud. Okay? Go for it.

Student: Done. Well it says there is part a, part b and part c so I labeled them. *[Step 3]* In the first section I did the question what is the relay's team median practice time. *[Step 1]* The median was the middle number and so I had to put the data in order cuz if I were to do it just like that it wouldn't come out right(*step 5*) so I had to put them in order. *[Steps 1 and 3]* Then I crossed out them until I got to the middle and I got 3.39. *[Step 4]* In part b is says, what is the relay team's average time and I put Wednesday, 3.89 because it was the most and the average means like the most, the biggest. *[Steps 1, 3, and 4]* Then in part c, it says is there an outlier in this data and I put 3.89 because it's the number that is the farthest from all of them. *[Steps 1, 3, and 4]*

Teacher: Could the outlier be 3.14?

Student: Well, 3.24 is close to 3.14 and 3.48 isn't that close to 3.89 so no the outlier couldn't be 3.14.

Teacher: Here is another problem. It has to do with the same kind of thing. I would like you to read it and explain and write down anything thinking you had and explain it to me after that. Okay?

Student: For this one it was also a part a, part b, part c problem and I labeled them part a, part b, part c. *[Step 1]* Then the first question was what was the mean average high temperature for the five days and I put Tuesday, 93 degrees because on the high days, Tuesday was the hottest day in the week. *[Steps 1, 2, 3, and 4]* Part b was what was the median low temperature of the five days. *[Step 1]* For this one I had to also arrange them from the lowest to the highest and then cross them our from the ends and see which one is in the middle. *[Step 3]* Then for part c, it was what was the mode for the ten temperatures. I put 16 because mode means to take the highest number and you subtract the lowest number. *[Steps 1, 3, and 4]* Oh, wait, I put 79 as the highest but I needed to put 93 because I only looked in the low column so the mode should be 30 degrees *[Step 5]*

Esmeralda's Recording:

Teacher: Okay, so what I want you to do is read the problem carefully and think about any steps that you find that you think are important and write those down. Then you say okay, I'm ready, and explain to me what you are thinking out loud.

Student: Okay. Done.

Teacher: Okay, so explain to me what you did.

Student: Part a, I wrote all the numbers in order and then I did a slash on them from either side cuz it says find the median in the middle. *[Steps 1, 2, and 3]* I got 3.39. *[Step 4]*

Teacher: Alright, what else?

Student: Part b, I organized the big number to the small number but I wasn't sure what average meant. So I tried to subtract the biggest number minus the smallest number cuz I remember that from math. *[Steps 1, 3, and 6]*

Teacher: Okay, how about part c?

Student: In part c they asked about the outlier and I said yeah because 3.89 is far away from all of the other numbers. *[Steps 1, 2, 3, and 4]* Since 3.89 is the biggest number away from the others, I picked it. *[Steps 3 and 4]*

Teacher: Is the outlier always the biggest number?

Student: No it can be the smallest number too but since there is a bigger gap between 3.89 and 3.48 than 3.14 and 3.24, 3.89 has to be the outlier. *[Step 6]*

Teacher: Awesome. Okay so just one more problem. I want you to do the same thing. You should read it, make sure you see any key words, figure out what you have to do to solve it and then explain it to me.

Student: Okay. Done.

Teacher: Okay. Go ahead and explain to me what you did.

Student: For part a, I minused 93 minus 88 and I got five because I know that mean means average. *[Steps 1, 3, and 4]* I took the highest number and subtracted the smallest from that. *[Step 3]* Oh, wait. The lowest number is supposed to be 85 so I need to fix my answer. *[Step 5]*

Teacher: Okay, good.

Student: For part b it says find the median so I ordered the numbers from small to big and then crossed out and I got 88 in the middle. *[Steps 1, 2, 3, and 4]*

Teacher: Okay, and the last part.

Student: For part c, it says find the mode and it is the most. I remember that from fifth grade. I knew I had to look at all of the ten temperatures and I got 79 because there are three 79s and only one of the other numbers. *[Steps 1, 2, 3, and 4]*

[Steps 1, 2, 3, 4, and 6 used]

Melissa's Recording:

Teacher: Today I have two word problems for you. Both of them are about mean, median mode and range. So you are going to read it, you can write any steps that you want to, pull out any words that you feel have specific meaning and explain it to me out loud.

Student: Okay. Ready.

Teacher: Okay, explain it to me.

Student: For part a, it said what is the relay team's median practice time. I crossed out the numbers *[Step 4]* and then the median one is 3.39. First I put them in order. *[Step 3]* Then in part b, I add all of the times *[Step 3]* because it says average *[Step 2]* and then I got 17.4 cuz I add all of them. In part C, it said what is the outlier *[Step 1]*. I said 3.89 is the outlier because it's like out and far away from the other numbers *[Step 3]* and 3.14 isn't the outlier because it isn't as far away from the other numbers *[Step 5]*.

Teacher: Okay, we have one more problem. You're gonna read it and then tell me what you did.

Student: Okay.

Teacher: Go ahead and explain it.

Student: Well, first I put part a, part b and part c. Then it said what is the mean *[Step 1]* and I got 88 cuz I just put the numbers in order and then started crossing out until I got to the middle. *[Steps 3 and 4]* I used the numbers in the high temperature column because that's what the question said. *[Steps 1 and 2]*

In part b, it said low temperatures, so I did the same thing but with the low temperatures. I put the numbers in order and I crossed out from each end until I got 68 cuz that was the middle one. *[Steps 1, 2, 3, and 4]*

For part c, it said to look at the two tables and then I just put them in order and then I subtracted 93 minus 63 and it equals 30 degrees. *[Steps 3 and 4]* I did that cuz it says the mode and the mode means subtract the highest and the lowest numbers. *[Steps 1 and 2]*

[Steps 1,2,3,4, and 5 used]

Apple's Recording:

Teacher: Okay, so you are going to read the problem and you are going to write down anything that you did when you were thinking about it. Then you will stop and explain it to me.

Student: Okay. First, I put them in order so I could get the right median and then I just crossed out this and the other side to get the middle. *[Steps 3 and 4]*

Next, I add them all up because it's the mean or average *[Step 1]* and I divided them by the number of numbers there were. So because there were five numbers, I divided by five and I got 3.428. *[Steps 3 and 4]*

Teacher: Awesome, how about the last one?

Student: Well, all these numbers are close to each other except for this last one, 3.89 is way far. *[Steps 3 and 6]* Since it said outlier I know that means like out of the place, like way far away from the others. *[Steps 1, 2, 3, and 4]*

Teacher: Okay, so, you are gonna read this, write down all of your steps and then tell me what you are thinking once you are done. Go for it.

Student: Okay. For part a, it asked for the high temperatures, the mean, *[Step 1]* so I just put the numbers all in order and added them all up because the mean means add them and divide by the number of numbers you have. *[Steps 2, 3, and 4]* Then part b, it asked for the low and I put those in order and then I crossed off the sides until I got to the middle *[Steps 3, 4, and 6]* cuz the median means the middle. *[Steps 1 and 2]* In the last part, it asks for all the ten temperatures, so it asked for the mode, which is most often, and I found 79 because there are three. *[Steps 1, 2, 3, and 4]*

[Steps 1, 2, 3, 4, 5, and 6 used]

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